

OEIS A181189

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ABSTRACT. This is a list of the statistics of the orders of the elements for all Abelian Groups up to group order 64, and the GAP program generating this information.

1. ELEMENT ORDERS IN ABELIAN GROUPS

Sequence A181189 of the OEIS [1] asks for the worst case of collecting element orders of an unknown Abelian Group (of known group order) until the list of element orders gathered randomly identifies the Abelian Group uniquely.

The following is a list of groups G_i of increasing order. The first integer in a line is the group order $|G|$, and the second integer the index i of the group assigned (in some arbitrary way) by GAP [2] up to the number of groups [1, A000001], and the rest of a line a notation for the group structure (using for example C for cyclic groups, D for dihedral groups, the x for direct products etc.)

If that group is abelian, another line (starting with a gap) follows with a list of numbers of the form o^f which indicates that f elements of that group have order o . The sum of these frequencies is $\sum f = |G|$, the group order.

The number of these extra lines for each group order is in [1, A000688].

Groups of order 1

1 1 1
1¹

Groups of order 2

2 1 C2
1¹ 2¹

Groups of order 3

3 1 C3
1¹ 3²

Groups of order 4

4 1 C4
1¹ 2¹ 4²
4 2 C2 x C2
1¹ 2³

Groups of order 5

5 1 C5

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$$1^1 5^4$$

Groups of order 6

6 1 S3

6 2 C6

$$1^1 2^1 3^2 6^2$$

Groups of order 7

7 1 C7

$$1^1 7^6$$

Groups of order 8

8 1 C8

$$1^1 2^1 4^2 8^4$$

8 2 C4 x C2

$$1^1 2^3 4^4$$

8 3 D8

8 4 Q8

8 5 C2 x C2 x C2

$$1^1 2^7$$

Groups of order 9

9 1 C9

$$1^1 3^2 9^6$$

9 2 C3 x C3

$$1^1 3^8$$

Groups of order 10

10 1 D10

10 2 C10

$$1^1 2^1 5^4 10^4$$

Groups of order 11

11 1 C11

$$1^1 11^{10}$$

Groups of order 12

12 1 C3 : C4

12 2 C12

$$1^1 2^1 3^2 4^2 6^2 12^4$$

12 3 A4

12 4 D12

12 5 C6 x C2

$$1^1 2^3 3^2 6^6$$

Groups of order 13

13 1 C13

$$1^1 13^{12}$$

Groups of order 14

14 1 D14

14 2 C14

$$1^1 2^1 7^6 14^6$$

Groups of order 15

15 1 C15
 $1^1 3^2 5^4 15^8$

Groups of order 16

16 1 C16
 $1^1 2^1 4^2 8^4 16^8$
 16 2 C4 x C4
 $1^1 2^3 4^{12}$
 16 3 (C4 x C2) : C2
 16 4 C4 : C4
 16 5 C8 x C2
 $1^1 2^3 4^4 8^8$
 16 6 C8 : C2
 16 7 D16
 16 8 QD16
 16 9 Q16
 16 10 C4 x C2 x C2
 $1^1 2^7 4^8$
 16 11 C2 x D8
 16 12 C2 x Q8
 16 13 (C4 x C2) : C2
 16 14 C2 x C2 x C2 x C2
 $1^1 2^{15}$

Groups of order 17

17 1 C17
 $1^1 17^{16}$

Groups of order 18

18 1 D18
 18 2 C18
 $1^1 2^1 3^2 6^2 9^6 18^6$
 18 3 C3 x S3
 18 4 (C3 x C3) : C2
 18 5 C6 x C3
 $1^1 2^1 3^8 6^8$

Groups of order 19

19 1 C19
 $1^1 19^{18}$

Groups of order 20

20 1 C5 : C4
 20 2 C20
 $1^1 2^1 4^2 5^4 10^4 20^8$
 20 3 C5 : C4
 20 4 D20
 20 5 C10 x C2
 $1^1 2^3 5^4 10^{12}$

Groups of order 21

21 1 C7 : C3
 21 2 C21
 $1^1 3^2 7^6 21^{12}$

Groups of order 22

22 1 D22
 22 2 C22
 $1^1 2^1 11^{10} 22^{10}$

Groups of order 23

23 1 C23
 $1^1 23^{22}$

Groups of order 24

24 1 C3 : C8
 24 2 C24
 $1^1 2^1 3^2 4^2 6^2 8^4 12^4 24^8$
 24 3 SL(2,3)
 24 4 C3 : Q8
 24 5 C4 x S3
 24 6 D24
 24 7 C2 x (C3 : C4)
 24 8 (C6 x C2) : C2
 24 9 C12 x C2
 $1^1 2^3 3^2 4^4 6^6 12^8$
 24 10 C3 x D8
 24 11 C3 x Q8
 24 12 S4
 24 13 C2 x A4
 24 14 C2 x C2 x S3
 24 15 C6 x C2 x C2
 $1^1 2^7 3^2 6^{14}$

Groups of order 25

25 1 C25
 $1^1 5^4 25^{20}$
 25 2 C5 x C5
 $1^1 5^{24}$

Groups of order 26

26 1 D26
 26 2 C26
 $1^1 2^1 13^{12} 26^{12}$

Groups of order 27

27 1 C27
 $1^1 3^2 9^6 27^{18}$
 27 2 C9 x C3
 $1^1 3^8 9^{18}$
 27 3 (C3 x C3) : C3
 27 4 C9 : C3
 27 5 C3 x C3 x C3
 $1^1 3^{26}$

Groups of order 28

28 1 C7 : C4

28 2 C28

$$1^1 2^1 4^2 7^6 14^6 28^{12}$$

28 3 D28

28 4 C14 x C2

$$1^1 2^3 7^6 14^{18}$$

Groups of order 29

29 1 C29

$$1^1 29^{28}$$

Groups of order 30

30 1 C5 x S3

30 2 C3 x D10

30 3 D30

30 4 C30

$$1^1 2^1 3^2 5^4 6^2 10^4 15^8 30^8$$

Groups of order 31

31 1 C31

$$1^1 31^{30}$$

Groups of order 32

32 1 C32

$$1^1 2^1 4^2 8^4 16^8 32^{16}$$

32 2 (C4 x C2) : C4

32 3 C8 x C4

$$1^1 2^3 4^{12} 8^{16}$$

32 4 C8 : C4

32 5 (C8 x C2) : C2

32 6 (C2 x C2 x C2) : C4

32 7 (C8 : C2) : C2

32 8 C2 . ((C4 x C2) : C2) = (C2 x C2) . (C4 x C2)

32 9 (C8 x C2) : C2

32 10 Q8 : C4

32 11 (C4 x C4) : C2

32 12 C4 : C8

32 13 C8 : C4

32 14 C8 : C4

32 15 C4 . D8 = C4 . (C4 x C2)

32 16 C16 x C2

$$1^1 2^3 4^4 8^8 16^{16}$$

32 17 C16 : C2

32 18 D32

32 19 QD32

32 20 Q32

32 21 C4 x C4 x C2

$$1^1 2^7 4^{24}$$

32 22 C2 x ((C4 x C2) : C2)

32 23 C2 x (C4 : C4)

32 24 (C4 x C4) : C2

32 25 $C_4 \times D_8$
 32 26 $C_4 \times Q_8$
 32 27 $(C_2 \times C_2 \times C_2 \times C_2) : C_2$
 32 28 $(C_4 \times C_2 \times C_2) : C_2$
 32 29 $(C_2 \times Q_8) : C_2$
 32 30 $(C_4 \times C_2 \times C_2) : C_2$
 32 31 $(C_4 \times C_4) : C_2$
 32 32 $(C_2 \times C_2) \cdot (C_2 \times C_2 \times C_2)$
 32 33 $(C_4 \times C_4) : C_2$
 32 34 $(C_4 \times C_4) : C_2$
 32 35 $C_4 : Q_8$
 32 36 $C_8 \times C_2 \times C_2$
 $1^1 2^7 4^8 8^{16}$
 32 37 $C_2 \times (C_8 : C_2)$
 32 38 $(C_8 \times C_2) : C_2$
 32 39 $C_2 \times D_{16}$
 32 40 $C_2 \times QD_{16}$
 32 41 $C_2 \times Q_{16}$
 32 42 $(C_8 \times C_2) : C_2$
 32 43 $C_8 : (C_2 \times C_2)$
 32 44 $(C_2 \times Q_8) : C_2$
 32 45 $C_4 \times C_2 \times C_2 \times C_2$
 $1^1 2^{15} 4^{16}$
 32 46 $C_2 \times C_2 \times D_8$
 32 47 $C_2 \times C_2 \times Q_8$
 32 48 $C_2 \times ((C_4 \times C_2) : C_2)$
 32 49 $(C_2 \times C_2 \times C_2) : (C_2 \times C_2)$
 32 50 $(C_2 \times Q_8) : C_2$
 32 51 $C_2 \times C_2 \times C_2 \times C_2 \times C_2$
 $1^1 2^{31}$

Groups of order 33

33 1 C_{33}
 $1^1 3^2 11^{10} 33^{20}$

Groups of order 34

34 1 D_{34}
 34 2 C_{34}
 $1^1 2^1 17^{16} 34^{16}$

Groups of order 35

35 1 C_{35}
 $1^1 5^4 7^6 35^{24}$

Groups of order 36

36 1 $C_9 : C_4$
 36 2 C_{36}
 $1^1 2^1 3^2 4^2 6^2 9^6 12^4 18^6 36^{12}$
 36 3 $(C_2 \times C_2) : C_9$
 36 4 D_{36}
 36 5 $C_{18} \times C_2$
 $1^1 2^3 3^2 6^6 9^6 18^{18}$
 36 6 $C_3 \times (C_3 : C_4)$

36 7 (C3 x C3) : C4
 36 8 C12 x C3
 $1^1 2^1 3^8 4^2 6^8 12^{16}$
 36 9 (C3 x C3) : C4
 36 10 S3 x S3
 36 11 C3 x A4
 36 12 C6 x S3
 36 13 C2 x ((C3 x C3) : C2)
 36 14 C6 x C6
 $1^1 2^3 3^8 6^{24}$

Groups of order 37

37 1 C37
 $1^1 37^{36}$

Groups of order 38

38 1 D38
 38 2 C38
 $1^1 2^1 19^{18} 38^{18}$

Groups of order 39

39 1 C13 : C3
 39 2 C39
 $1^1 3^2 13^{12} 39^{24}$

Groups of order 40

40 1 C5 : C8
 40 2 C40
 $1^1 2^1 4^2 5^4 8^4 10^4 20^8 40^{16}$
 40 3 C5 : C8
 40 4 C5 : Q8
 40 5 C4 x D10
 40 6 D40
 40 7 C2 x (C5 : C4)
 40 8 (C10 x C2) : C2
 40 9 C20 x C2
 $1^1 2^3 4^4 5^4 10^{12} 20^{16}$
 40 10 C5 x D8
 40 11 C5 x Q8
 40 12 C2 x (C5 : C4)
 40 13 C2 x C2 x D10
 40 14 C10 x C2 x C2
 $1^1 2^7 5^4 10^{28}$

Groups of order 41

41 1 C41
 $1^1 41^{40}$

Groups of order 42

42 1 C7 : C6
 42 2 C2 x (C7 : C3)
 42 3 C7 x S3
 42 4 C3 x D14

42 5 D42

42 6 C42

$1^1 2^1 3^2 6^2 7^6 14^6 21^{12} 42^{12}$

Groups of order 43

43 1 C43

$1^1 43^{42}$

Groups of order 44

44 1 C11 : C4

44 2 C44

$1^1 2^1 4^2 11^{10} 22^{10} 44^{20}$

44 3 D44

44 4 C22 x C2

$1^1 2^3 11^{10} 22^{30}$

Groups of order 45

45 1 C45

$1^1 3^2 5^4 9^6 15^8 45^{24}$

45 2 C15 x C3

$1^1 3^8 5^4 15^{32}$

Groups of order 46

46 1 D46

46 2 C46

$1^1 2^1 23^{22} 46^{22}$

Groups of order 47

47 1 C47

$1^1 47^{46}$

Groups of order 48

48 1 C3 : C16

48 2 C48

$1^1 2^1 3^2 4^2 6^2 8^4 12^4 16^8 24^8 48^{16}$

48 3 (C4 x C4) : C3

48 4 C8 x S3

48 5 C24 : C2

48 6 C24 : C2

48 7 D48

48 8 C3 : Q16

48 9 C2 x (C3 : C8)

48 10 (C3 : C8) : C2

48 11 C4 x (C3 : C4)

48 12 (C3 : C4) : C4

48 13 C12 : C4

48 14 (C12 x C2) : C2

48 15 (C3 x D8) : C2

48 16 (C3 : Q8) : C2

48 17 (C3 x Q8) : C2

48 18 C3 : Q16

48 19 (C6 x C2) : C4

48 20 C12 x C4

$1^1 2^3 3^2 4^{12} 6^6 12^{24}$
 48 21 $C_3 \times ((C_4 \times C_2) : C_2)$
 48 22 $C_3 \times (C_4 : C_4)$
 48 23 $C_{24} \times C_2$
 $1^1 2^3 3^2 4^4 6^6 8^8 12^8 24^{16}$
 48 24 $C_3 \times (C_8 : C_2)$
 48 25 $C_3 \times D_{16}$
 48 26 $C_3 \times QD_{16}$
 48 27 $C_3 \times Q_{16}$
 48 28 $C_2 \cdot S_4 = SL(2,3) \cdot C_2$
 48 29 $GL(2,3)$
 48 30 $A_4 : C_4$
 48 31 $C_4 \times A_4$
 48 32 $C_2 \times SL(2,3)$
 48 33 $((C_4 \times C_2) : C_2) : C_3$
 48 34 $C_2 \times (C_3 : Q_8)$
 48 35 $C_2 \times C_4 \times S_3$
 48 36 $C_2 \times D_{24}$
 48 37 $(C_{12} \times C_2) : C_2$
 48 38 $D_8 \times S_3$
 48 39 $(C_4 \times S_3) : C_2$
 48 40 $Q_8 \times S_3$
 48 41 $(C_4 \times S_3) : C_2$
 48 42 $C_2 \times C_2 \times (C_3 : C_4)$
 48 43 $C_2 \times ((C_6 \times C_2) : C_2)$
 48 44 $C_{12} \times C_2 \times C_2$
 $1^1 2^7 3^2 4^8 6^{14} 12^{16}$
 48 45 $C_6 \times D_8$
 48 46 $C_6 \times Q_8$
 48 47 $C_3 \times ((C_4 \times C_2) : C_2)$
 48 48 $C_2 \times S_4$
 48 49 $C_2 \times C_2 \times A_4$
 48 50 $(C_2 \times C_2 \times C_2 \times C_2) : C_3$
 48 51 $C_2 \times C_2 \times C_2 \times S_3$
 48 52 $C_6 \times C_2 \times C_2 \times C_2$
 $1^1 2^{15} 3^2 6^{30}$

Groups of order 49

49 1 C_{49}
 $1^1 7^6 49^{42}$
 49 2 $C_7 \times C_7$
 $1^1 7^{48}$

Groups of order 50

50 1 D_{50}
 50 2 C_{50}
 $1^1 2^1 5^4 10^4 25^{20} 50^{20}$
 50 3 $C_5 \times D_{10}$
 50 4 $(C_5 \times C_5) : C_2$
 50 5 $C_{10} \times C_5$
 $1^1 2^1 5^{24} 10^{24}$

Groups of order 51

51 1 C51

$1^1 3^2 17^{16} 51^{32}$

Groups of order 52

52 1 C13 : C4

52 2 C52

$1^1 2^1 4^2 13^{12} 26^{12} 52^{24}$

52 3 C13 : C4

52 4 D52

52 5 C26 x C2

$1^1 2^3 13^{12} 26^{36}$

Groups of order 53

53 1 C53

$1^1 53^{52}$

Groups of order 54

54 1 D54

54 2 C54

$1^1 2^1 3^2 6^2 9^6 18^6 27^{18} 54^{18}$

54 3 C3 x D18

54 4 C9 x S3

54 5 (C3 x C3) : C6

54 6 C9 : C6

54 7 (C9 x C3) : C2

54 8 ((C3 x C3) : C3) : C2

54 9 C18 x C3

$1^1 2^1 3^8 6^8 9^{18} 18^{18}$

54 10 C2 x ((C3 x C3) : C3)

54 11 C2 x (C9 : C3)

54 12 C3 x C3 x S3

54 13 C3 x ((C3 x C3) : C2)

54 14 (C3 x C3 x C3) : C2

54 15 C6 x C3 x C3

$1^1 2^1 3^{26} 6^{26}$

Groups of order 55

55 1 C11 : C5

55 2 C55

$1^1 5^4 11^{10} 55^{40}$

Groups of order 56

56 1 C7 : C8

56 2 C56

$1^1 2^1 4^2 7^6 8^4 14^6 28^{12} 56^{24}$

56 3 C7 : Q8

56 4 C4 x D14

56 5 D56

56 6 C2 x (C7 : C4)

56 7 (C14 x C2) : C2

56 8 C28 x C2

$1^1 2^3 4^4 7^6 14^{18} 28^{24}$

56 9 C7 x D8

56 10 C7 x Q8
 56 11 (C2 x C2 x C2) : C7
 56 12 C2 x C2 x D14
 56 13 C14 x C2 x C2
 $1^1 2^7 7^6 14^{42}$

Groups of order 57
 57 1 C19 : C3
 57 2 C57
 $1^1 3^2 19^{18} 57^{36}$

Groups of order 58
 58 1 D58
 58 2 C58
 $1^1 2^1 29^{28} 58^{28}$

Groups of order 59
 59 1 C59
 $1^1 59^{58}$

Groups of order 60
 60 1 C5 x (C3 : C4)
 60 2 C3 x (C5 : C4)
 60 3 C15 : C4
 60 4 C60
 $1^1 2^1 3^2 4^2 5^4 6^2 10^4 12^4 15^8 20^8 30^8 60^{16}$
 60 5 A5
 60 6 C3 x (C5 : C4)
 60 7 C15 : C4
 60 8 S3 x D10
 60 9 C5 x A4
 60 10 C6 x D10
 60 11 C10 x S3
 60 12 D60
 60 13 C30 x C2
 $1^1 2^3 3^2 5^4 6^6 10^{12} 15^8 30^{24}$

Groups of order 61
 61 1 C61
 $1^1 61^{60}$

Groups of order 62
 62 1 D62
 62 2 C62
 $1^1 2^1 31^{30} 62^{30}$

Groups of order 63
 63 1 C7 : C9
 63 2 C63
 $1^1 3^2 7^6 9^6 21^{12} 63^{36}$
 63 3 C3 x (C7 : C3)
 63 4 C21 x C3
 $1^1 3^8 7^6 21^{48}$

Groups of order 64

64 1 C64
 $1^1 2^1 4^2 8^4 16^8 32^{16} 64^{32}$
 64 2 C8 x C8
 $1^1 2^3 4^{12} 8^{48}$
 64 3 C8 : C8
 64 4 ((C8 x C2) : C2) : C2
 64 5 (C4 x C2) : C8
 64 6 (C8 x C4) : C2
 64 7 Q8 : C8
 64 8 ((C8 x C2) : C2) : C2
 64 9 (C4 : C4) : C4
 64 10 (C8 : C4) : C2
 64 11 (C2 x C2) . ((C4 x C2) : C2) = (C4 x C2) . (C4 x C2)
 64 12 (C4 : C8) : C2
 64 13 (C2 x C2) . ((C4 x C2) : C2) = (C4 x C2) . (C4 x C2)
 64 14 (C2 x C2) . ((C4 x C2) : C2) = (C4 x C2) . (C4 x C2)
 64 15 C8 : C8
 64 16 C8 : C8
 64 17 (C8 x C2) : C4
 64 18 (C4 x C4) : C4
 64 19 C4 . (C4 x C4)
 64 20 (C4 : C4) : C4
 64 21 (C4 : C4) : C4
 64 22 (C4 x C2) . D8 = C4 . (C4 x C4)
 64 23 ((C4 x C2) : C2) : C4
 64 24 (C8 : C2) : C4
 64 25 (C8 x C2) : C4
 64 26 C16 x C4
 $1^1 2^3 4^{12} 8^{16} 16^{32}$
 64 27 C16 : C4
 64 28 C16 : C4
 64 29 (C16 x C2) : C2
 64 30 (C16 : C2) : C2
 64 31 (C16 : C2) : C2
 64 32 ((C8 : C2) : C2) : C2
 64 33 (C4 x C2 x C2) : C4
 64 34 ((C2 x C2 x C2) : C4) : C2
 64 35 (C2 x Q8) : C4
 64 36 (C2 . ((C4 x C2) : C2) = (C2 x C2) . (C4 x C2)) : C2
 64 37 C2 . ((C2 x C2 x C2) : C4) = (C4 x C2) . (C4 x C2)
 64 38 (C16 x C2) : C2
 64 39 Q16 : C4
 64 40 (C16 x C2) : C2
 64 41 (C16 : C2) : C2
 64 42 (C16 : C2) : C2
 64 43 C2 . ((C8 x C2) : C2) = C8 . (C4 x C2)
 64 44 C4 : C16
 64 45 C8 . D8 = C4 . (C8 x C2)
 64 46 C16 : C4
 64 47 C16 : C4
 64 48 C16 : C4

64 49 $C4 \cdot D16 = C8 \cdot (C4 \times C2)$
 64 50 $C32 \times C2$
 $1^1 2^3 4^4 8^8 16^{16} 32^{32}$
 64 51 $C32 : C2$
 64 52 $D64$
 64 53 $QD64$
 64 54 $Q64$
 64 55 $C4 \times C4 \times C4$
 $1^1 2^7 4^{56}$
 64 56 $C2 \times ((C4 \times C2) : C4)$
 64 57 $(C4 \times C4) : C4$
 64 58 $C4 \times ((C4 \times C2) : C2)$
 64 59 $C4 \times (C4 : C4)$
 64 60 $(C2 \times ((C4 \times C2) : C2)) : C2$
 64 61 $((C4 \times C2) : C4) : C2$
 64 62 $((C4 \times C2) : C4) : C2$
 64 63 $(C4 \times C4) : C4$
 64 64 $(C4 \times C4) : C4$
 64 65 $(C4 \times C4) : C4$
 64 66 $(C2 \times (C4 : C4)) : C2$
 64 67 $(C2 \times ((C4 \times C2) : C2)) : C2$
 64 68 $(C4 : C4) : C4$
 64 69 $((C4 \times C2) : C4) : C2$
 64 70 $(C4 : C4) : C4$
 64 71 $(C2 \times (C4 : C4)) : C2$
 64 72 $(C2 \times Q8) : C4$
 64 73 $(C2 \times C2 \times D8) : C2$
 64 74 $((C4 \times C2) : C4) : C2$
 64 75 $(C2 \times ((C4 \times C2) : C2)) : C2$
 64 76 $(C4 \times C2) : Q8$
 64 77 $(C2 \times (C4 : C4)) : C2$
 64 78 $(C2 \times (C4 : C4)) : C2$
 64 79 $(C2 \times C2 \times C2) \cdot (C2 \times C2 \times C2)$
 64 80 $((C4 \times C2) : C4) : C2$
 64 81 $(C2 \times C2 \times C2) \cdot (C2 \times C2 \times C2)$
 64 82 $(C2 \times C2 \times C2) \cdot (C2 \times C2 \times C2)$
 64 83 $C8 \times C4 \times C2$
 $1^1 2^7 4^{24} 8^{32}$
 64 84 $C2 \times (C8 : C4)$
 64 85 $C4 \times (C8 : C2)$
 64 86 $(C8 \times C4) : C2$
 64 87 $C2 \times ((C8 \times C2) : C2)$
 64 88 $(C2 \times (C8 : C2)) : C2$
 64 89 $(C8 \times C2 \times C2) : C2$
 64 90 $C2 \times ((C2 \times C2 \times C2) : C4)$
 64 91 $((C4 \times C4) : C2) : C2$
 64 92 $C2 \times ((C8 : C2) : C2)$
 64 93 $C2 \times (C2 \cdot ((C4 \times C2) : C2) = (C2 \times C2) \cdot (C4 \times C2))$
 64 94 $(C2 \times (C8 : C2)) : C2$
 64 95 $C2 \times ((C8 \times C2) : C2)$
 64 96 $C2 \times (Q8 : C4)$
 64 97 $((C4 \times C4) : C2) : C2$
 64 98 $(C2 \times (C4 : C4)) : C2$

64 99 $((C4 \times C4) : C2) : C2$
 64 100 $(Q8 : C4) : C2$
 64 101 $C2 \times ((C4 \times C4) : C2)$
 64 102 $(C2 \times (C8 : C2)) : C2$
 64 103 $C2 \times (C4 : C8)$
 64 104 $(C4 : C8) : C2$
 64 105 $(C4 : C8) : C2$
 64 106 $C2 \times (C8 : C4)$
 64 107 $C2 \times (C8 : C4)$
 64 108 $(C8 : C4) : C2$
 64 109 $(C8 : C4) : C2$
 64 110 $C2 \times (C4 \cdot D8 = C4 \cdot (C4 \times C2))$
 64 111 $(C4 \cdot D8 = C4 \cdot (C4 \times C2)) : C2$
 64 112 $(C8 \times C4) : C2$
 64 113 $(C8 : C4) : C2$
 64 114 $(C8 \times C4) : C2$
 64 115 $C8 \times D8$
 64 116 $(C2 \times (C8 : C2)) : C2$
 64 117 $(C2 \times (C8 : C2)) : C2$
 64 118 $C4 \times D16$
 64 119 $C4 \times QD16$
 64 120 $C4 \times Q16$
 64 121 $(C4 \times Q8) : C2$
 64 122 $Q16 : C4$
 64 123 $(C4 \times D8) : C2$
 64 124 $((C8 \times C2) : C2) : C2$
 64 125 $((C8 \times C2) : C2) : C2$
 64 126 $C8 \times Q8$
 64 127 $C8 : Q8$
 64 128 $((C4 \times C2 \times C2) : C2) : C2$
 64 129 $(C2 \times QD16) : C2$
 64 130 $((C4 \times C2 \times C2) : C2) : C2$
 64 131 $((C2 \times Q8) : C2) : C2$
 64 132 $(C2 \times Q16) : C2$
 64 133 $((C2 \times Q8) : C2) : C2$
 64 134 $((C4 \times C4) : C2) : C2$
 64 135 $((C4 \times C4) : C2) : C2$
 64 136 $((C4 \times C4) : C2) : C2$
 64 137 $(C4 : Q8) : C2$
 64 138 $((C2 \times C2 \times C2 \times C2) : C2) : C2$
 64 139 $((C4 \times C2 \times C2) : C2) : C2$
 64 140 $(C2 \times D16) : C2$
 64 141 $(C2 \times QD16) : C2$
 64 142 $(Q8 : C4) : C2$
 64 143 $C4 : Q16$
 64 144 $((C8 \times C2) : C2) : C2$
 64 145 $(C4 \times Q8) : C2$
 64 146 $((C8 \times C2) : C2) : C2$
 64 147 $((C4 \times C2 \times C2) : C2) : C2$
 64 148 $(Q8 : C4) : C2$
 64 149 $((C8 \times C2) : C2) : C2$
 64 150 $((C4 \times C2 \times C2) : C2) : C2$
 64 151 $(Q8 : C4) : C2$

64 152 ((C8 : C2) : C2) : C2
 64 153 (C8 : (C2 x C2)) : C2
 64 154 (C2 . ((C4 x C2) : C2) = (C2 x C2) . (C4 x C2)) : C2
 64 155 (C8 : C4) : C2
 64 156 Q8 : Q8
 64 157 (C8 : C4) : C2
 64 158 Q8 : Q8
 64 159 (C8 : C4) : C2
 64 160 (C2 x C2) . (C2 x D8) = (C4 x C2) . (C2 x C2 x C2)
 64 161 ((C8 x C2) : C2) : C2
 64 162 ((C8 x C2) : C2) : C2
 64 163 ((C8 x C2) : C2) : C2
 64 164 (Q8 : C4) : C2
 64 165 (Q8 : C4) : C2
 64 166 (Q8 : C4) : C2
 64 167 ((C8 x C2) : C2) : C2
 64 168 (C2 x C2) . (C2 x D8) = (C4 x C2) . (C2 x C2 x C2)
 64 169 (C8 x C4) : C2
 64 170 (C8 : C4) : C2
 64 171 ((C8 x C2) : C2) : C2
 64 172 (C2 x C2) . (C2 x D8) = (C4 x C2) . (C2 x C2 x C2)
 64 173 (C2 x QD16) : C2
 64 174 ((C4 x C4) : C2) : C2
 64 175 C4 : Q16
 64 176 ((C4 x C4) : C2) : C2
 64 177 ((C4 x C4) : C2) : C2
 64 178 (C4 : Q8) : C2
 64 179 C8 : Q8
 64 180 (C2 x C2) . (C2 x D8) = (C4 x C2) . (C2 x C2 x C2)
 64 181 C8 : Q8
 64 182 C8 : Q8
 64 183 C16 x C2 x C2
 1¹ 2⁷ 4⁸ 8¹⁶ 16³²
 64 184 C2 x (C16 : C2)
 64 185 (C16 x C2) : C2
 64 186 C2 x D32
 64 187 C2 x QD32
 64 188 C2 x Q32
 64 189 ((C8 x C2) : C2) : C2
 64 190 ((C8 x C2) : C2) : C2
 64 191 QD32 : C2
 64 192 C4 x C4 x C2 x C2
 1¹ 2¹⁵ 4⁴⁸
 64 193 C2 x C2 x ((C4 x C2) : C2)
 64 194 C2 x C2 x (C4 : C4)
 64 195 C2 x ((C4 x C4) : C2)
 64 196 C2 x C4 x D8
 64 197 C2 x C4 x Q8
 64 198 C4 x ((C4 x C2) : C2)
 64 199 (C2 x ((C4 x C2) : C2)) : C2
 64 200 (C4 x Q8) : C2
 64 201 ((C4 x C4) : C2) : C2
 64 202 C2 x ((C2 x C2 x C2 x C2) : C2)

64 203 $C2 \times ((C4 \times C2 \times C2) : C2)$
 64 204 $C2 \times ((C2 \times Q8) : C2)$
 64 205 $C2 \times ((C4 \times C2 \times C2) : C2)$
 64 206 $(C2 \times ((C4 \times C2) : C2)) : C2$
 64 207 $C2 \times ((C4 \times C4) : C2)$
 64 208 $C2 \times ((C2 \times C2) \cdot (C2 \times C2 \times C2))$
 64 209 $C2 \times ((C4 \times C4) : C2)$
 64 210 $((C4 \times C4) : C2) : C2$
 64 211 $C2 \times ((C4 \times C4) : C2)$
 64 212 $C2 \times (C4 : Q8)$
 64 213 $(C2 \times ((C4 \times C2) : C2)) : C2$
 64 214 $(C4 \times Q8) : C2$
 64 215 $(C2 \times C2 \times D8) : C2$
 64 216 $(C2 \times ((C4 \times C2) : C2)) : C2$
 64 217 $((C4 \times C4) : C2) : C2$
 64 218 $(C2 \times ((C4 \times C2) : C2)) : C2$
 64 219 $(C4 \times D8) : C2$
 64 220 $(C4 \times D8) : C2$
 64 221 $(C4 \times D8) : C2$
 64 222 $(C4 \times Q8) : C2$
 64 223 $(C4 \times D8) : C2$
 64 224 $((C2 \times Q8) : C2) : C2$
 64 225 $(C4 : Q8) : C2$
 64 226 $D8 \times D8$
 64 227 $(C2 \times ((C4 \times C2) : C2)) : C2$
 64 228 $(C2 \times ((C4 \times C2) : C2)) : C2$
 64 229 $(C2 \times ((C4 \times C2) : C2)) : C2$
 64 230 $Q8 \times D8$
 64 231 $(C2 \times ((C4 \times C2) : C2)) : C2$
 64 232 $((C4 \times C4) : C2) : C2$
 64 233 $((C4 \times C4) : C2) : C2$
 64 234 $((C4 \times C4) : C2) : C2$
 64 235 $(C2 \times (C4 : C4)) : C2$
 64 236 $((C4 \times C4) : C2) : C2$
 64 237 $((C4 \times C4) : C2) : C2$
 64 238 $Q8 : Q8$
 64 239 $Q8 \times Q8$
 64 240 $(C4 \times D8) : C2$
 64 241 $((C4 \times C2 \times C2) : C2) : C2$
 64 242 $((C4 \times C4) : C2) : C2$
 64 243 $((C4 \times C2 \times C2) : C2) : C2$
 64 244 $(C4 : Q8) : C2$
 64 245 $(C2 \times C2) \cdot (C2 \times C2 \times C2 \times C2)$
 64 246 $C8 \times C2 \times C2 \times C2$
 $1^1 2^{15} 4^{16} 8^{32}$
 64 247 $C2 \times C2 \times (C8 : C2)$
 64 248 $C2 \times ((C8 \times C2) : C2)$
 64 249 $(C2 \times (C8 : C2)) : C2$
 64 250 $C2 \times C2 \times D16$
 64 251 $C2 \times C2 \times QD16$
 64 252 $C2 \times C2 \times Q16$
 64 253 $C2 \times ((C8 \times C2) : C2)$
 64 254 $C2 \times (C8 : (C2 \times C2))$

```

64 255 C2 x ((C2 x Q8) : C2)
64 256 ((C8 x C2) : C2) : C2
64 257 (C2 x D16) : C2
64 258 (C2 x QD16) : C2
64 259 (C2 x Q16) : C2
64 260 C4 x C2 x C2 x C2 x C2
      1^1 2^31 4^32
64 261 C2 x C2 x C2 x D8
64 262 C2 x C2 x C2 x Q8
64 263 C2 x C2 x ((C4 x C2) : C2)
64 264 C2 x ((C2 x C2 x C2) : (C2 x C2))
64 265 C2 x ((C2 x Q8) : C2)
64 266 (C2 x ((C4 x C2) : C2)) : C2
64 267 C2 x C2 x C2 x C2 x C2 x C2
      1^1 2^63

```

The “worst” case of drawing elements from the unknown group is given by considering the smallest f for all orders o that appear in all Abelian Groups of the given order $|G|$. Example: for the two Abelian Groups of $|G| = 44$ one group has order statistics $1^1 2^1 4^2 11^{10} 22^{10} 44^{20}$ and the other has order statistics $1^1 2^3 11^{10} 22^{30}$. If the random sampling collects the unit element, one element of $o = 2$, 10 elements of $o = 11$, and 10 elements of $o = 22$ in any order (a total of $1+1+10+10 = 22$ samples), one still does not know whether G_2^{44} or G_4^{44} is present, so the worst number of samples is $1 + 22 = 23$.

2. GAP SOURCE CODE

The GAP program which generates that survey is [2]

```

#!/usr/bin/env gap

# print a list of frequencies of the group orders for group g.
gOrders := function(g)
  local els,divis,divisf,phi,e,o,posord ;
  # All elements of the group
  els := Elements(g) ;;

  # All divisors of the group order (i.e. potential
  # orders of the elements)
  divis := DivisorsInt(Order(g)) ;;

  # frequency (number of occurrences) of the orders;
  # initially a list of zeros.
  divisf := [] ;
  for phi in [1.. Length(divis)] do
    divisf[phi] := 0 ;
  od;

  for e in els do
    # run through all elements and store its order
    # to the divisf[] by increasing the frequency there
    o := Order(e) ;
    posord := Position(divis,o) ;

```

```

        divisf[posord] := divisf[posord]+1 ;
        # Print(" ",Order(e)) ;;
    od;
    Print("\n\t") ;;
    for phi in [1.. Length(divis)] do
        if divisf[phi] > 0 then
            Print(divis[phi],"^",divisf[phi]," ") ;
        fi;
    od;
end ;;

# for group g print the statistics of element orders
A181189 := function(g)
    local gAbel,gid;
    gAbel := 0 ;;
    gid := IdGroup(g) ;;
    Print(gid[1]," ",gid[2]) ;
    Print(" ",StructureDescription(g)) ;
    if IsAbelian(g) then
        gOrders(g) ;
        gAbel := gAbel+1 ;;
    fi ;
    Print("\n") ;
    return gAbel ;;
end;;

# run through all small groups of order 1,2,3...
for n in [1..64] do
    gAbel := 0 ;;
    Print("\nGroups of order ",n,"\n") ;
    alls := AllSmallGroups(n) ;
    for g in alls do
        gAbel := gAbel + A181189(g) ;
    od ;
    # Print how many of them were abelian
    # Print("\nAbelian :",gAbel,"\n") ;
od;
QUIT;

```

REFERENCES

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