n' = (MAXd+MINd)/2

Hello SeqFans,

Let's start with n = 723810.

We read n from left to right, digit by digit, in this way:

7 says: replace me by the biggest of the 7 digits on my right (including me)
Then 7 is replaced by 8 --> n = 823810

2 says: replace me by the biggest of the 2 digits on my right (including me)
Then 2 is replaced by 3 --> n = 833810

3 says: replace me by the biggest of the 3 digits on my right (including me)
Then 3 is replaced by 8 --> n = 838810

8 says: replace me by the biggest of the 8 digits on my right (including me)
Then 8 is replaced by 8 --> n = 838810

1 says: replace me by the biggest of the 1 digit(s) on my right (including me)
Then 1 is replaced by 1 --> n = 838810

0 says: replace me by the biggest of the 0 digit(s) on my right (including me)
Then 0 is replaced by nothing and disappears --> n = 83881

So n = 723810 becomes 83881. This result is called **MAXd** (maximum digit)

The **MINd** (minimum digit) operation works in the same way (just replace "biggest" by "smallest", above).

We then get for n: 723810 --> 02101 which is 2101.

We will now transform n into n':

n' = (MAXd+MINd)/2

For n = 723810 we have n' = (83881+2101)/2 = 42991

We could then iterate from there. But we will see first what happens with n = 1234 and present the iteration like this:

```
 / MAXd          / MAXd'
n  MAXd+MINd = n'  MAXd'+MINd' = n", etc.
 \ MINd          \ MINd'

    /1344       /1899       /1994       /1919       /1919
1234 2578 = 1289 3188 = 1594 3438 = 1719 3038 = 1519 3038 = 1519
    \1234       \1289       \1444       \1119       \1119
```

... we see that 1519 is a fixed point.

What would be **S**, the "fixed-points" sequence of (MAXd+MINd)/2?

S starts, I think, like this:

S = 1,2,3,4,5,6,7,8,9,11,12,13,14,15,16,17,18,19,22, ...

Could we have (MAXd+MINd) = odd number? Yes, in a (very) few cases:

```
  /3
30 3 = 1,5
  \0
```

If so, the iteration stops as "impossible".

Best,
É.

Douglas McNeil:

> **S** = 1,2,3,4,5,6,7,8,9,11,12,13,14,15,16,17,18,19,22, ...

Agreed: I find

sage: **S**
[1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 33, 44, 55, 66, 77, 88, 99, 111, 112, 113, 114, 115, 116, 117, 118, 119, 122, 133, 144, 155, 166, 177, 188, 199, 222, 315, 333, 417, 444, 519, 555, 666, 777, 888, 999, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1122, 1133, 1144, 1155, 1166, 1177, 1188, 1199, 1222, 1315, 1333, 1417, 1444, 1519, 1555, 1666, 1777, 1888, 1999, 2222, 3155, 3333, 3519, 4117, 4177, 4417, 4444, 5119, 5199, 5519, 5555, 6666, 7777, 8888, 9999]

> Could we have (MAXd+MINd) = odd number? Yes, in a (very) few cases:

Not so rare-- I think any multiple of 10 with a last nonzero digit being 3,5,7, or 9 will produce an odd number.

sage: **O**[:100]

[30, 50, 70, 90, 130, 150, 170, 190, 230, 250, 270, 290, 300, 330, 350, 370, 390, 430, 450, 470, 490, 500, 530, 550, 570, 590, 630, 650, 670, 690, 700, 730, 750, 770, 790, 830, 850, 870, 890, 900, 930, 950, 970, 990, 1030, 1050, 1070, 1090, 1130, 1150, 1170, 1190, 1230, 1250, 1270, 1290, 1300, 1330, 1350, 1370, 1390, 1430, 1450, 1470, 1490, 1500, 1530, 1550, 1570, 1590, 1630, 1650, 1670, 1690, 1700, 1730, 1750, 1770, 1790, 1830, 1850, 1870, 1890, 1900, 1930, 1950, 1970, 1990, 2030, 2050, 2070, 2090, 2130, 2150, 2170, 2190, 2230, 2250, 2270, 2290]

_____

Maximilian Hasler:

Je trouve pour les points fixes :

S =
1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 33, 44, 55, 66, 77, 88, 99, 111, 112, 113, 114, 115, 116, 117, 118, 119, 122, 133, 144, 155, 166, 177, 188, 199, 222, 315, 333, 417, 444, 519, 555, 666, 777, 888, 999, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1122, 1133, 1144, 1155, 1166, 1177, 1188, 1199, 1222, 1315, 1333, 1417, 1444, 1519, 1555, 1666, 1777, 1888, 1999, 2222, 3155, 3333, 3519, 4117, 4177, 4417, 4444, 5119, 5199, 5519, 5555, 6666, 7777, 8888, 9999, 11111, 11112, 11113, 11114, 11115, 11116, 11117, 11118, 11119, 11122, 11133, 11144, 11155, 11166, 11177, 11188, 11199, 11222, 1315, 11333, 11417, 11444, 11519, 11555, 11666, 11777, 11888, 11999, 12222, 13155, 13333, 13519, 14117, 14177, 14417, 14444, 15119, 15199, 15519, 15555, 16666, 17777, 18888, 19999, 22222, 31519, 31555, 33333, 35119, 35199, 41177, 41777, 44177, 44444, 51119, 51199, 51519, 51999, 53519, 55119, 55199, 55519, 55555, 66666, 77777, 88888, 99999, ...

... et pour les archives, le code :

```
MAXd(n)={ for(i=1,#n=Vecsmall(Str(n)), if( n[i]>48, for(j=i+1,min(#n,i+n[i]-49),
 n[j]>n[i] & n[i]=n[j]), n[i]=32)); eval(Strchr(n)) }

MINd(n)={ for(i=1,#n=Vecsmall(Str(n)), if( n[i]>48, for(j=i+1,min(#n,i+n[i]-49),
 n[j]<n[i] & n[i]=n[j]), n[i]=32)); eval(Strchr(n)) }

EA(n)=(MAXd(n)+MINd(n))/2

for(n=1,99999,EA(n)==n & print1(n", "))
```

_____

Jean-Marc Falcoz:

(...)
J'ai regardé ce que donnait la suite en allant jusqu'à 20 000 000 :

{1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 33, 44, 55, 66, 77, 88, 99, 111, 112, 113, 114, 115, 116, 117, 118, 119, 122, 133, 144, 155, 166, 177, 188, 199, 222, 315, 333, 417, 444, 519, 555, 666, 777, 888, 999, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1122, 1133, 1144, 1155, 1166, 1177, 1188, 1199, 1222, 1315, 1333, 1417, 1444, 1519, 1555, 1666, 1777, 1888, 1999, 2222, 3155, 3333, 3519, 4117, 4177, 4417, 4444, 5119, 5199, 5519, 5555, 6666, 7777, 8888, 9999, 11111, 11112, 11113, 11114, 11115, 11116, 11117, 11118, 11119, 11122, 11133, 11144, 11155, 11166, 11177, 11188, 11199, 11222, 11315, 11333, 11417, 11444, 11519, 11555, 11666, 11777, 11888, 11999, 12222, 13155, 13333, 13519, 14117, 14177, 14417, 14444, 15119, 15199, 15519, 15555, 16666, 17777, 18888, 19999, 22222, 31519, 31555, 33333, 35119, 35199, 41177, 41777, 44177, 44444, 51119, 51199, 51519, 51999, 53519, 55119, 55199, 55519, 55555, 66666, 77777, 88888, 99999, 111111, 111112, 111113, 111114, 111115, 111116, 111117, 111118, 111119, 111122, 111133, 111144, 111155, 111166, 111177, 111188, 111199, 111222, 111315, 111333, 111417, 111444, 111519, 111555, 111666, 111777, 111888, 111999, 112222, 113155, 113333, 113519, 114117, 114177, 114417, 114444, 115119, 115199, 115519, 115555, 116666, 117777, 118888, 119999, 122222, 131519, 131555, 133333, 135119, 135199, 141177, 141777, 144177, 144444, 151119, 151199, 151519, 151999, 153519, 155119, 155199, 155519, 155555, 166666, 177777, 188888, 199999, 222222, 315119, 315199, 315519, 315555, 333333, 351119, 351199, 351519, 351999, 411777, 417777, 441777, 444444, 453519, 511199, 511999, 515199, 519999, 535199, 551199, 551999, 555199, 555555, 666666, 777777, 888888, 999999, 1111111, 1111112, 1111113, 1111114, 1111115, 11111116, 11111117, 11111118, 11111119, 11111122, 11111133, 11111144, 11111155, 11111166, 11111177, 11111188, 11111199, 11111222, 11111315, 11111333, 11111417, 11111444, 11111519, 11111555, 11111666, 11111777, 11111888, 11111999, 11112222, 11113155, 11113333, 11113519, 11114117, 11114177, 11114417, 11114444, 11115119, 11115199, 11115519, 11115555, 11116666, 11117777, 11118888, 11119999, 11122222, 11131519, 11131555, 11133333, 11135119, 11135199, 11141177, 11141777, 11144177, 11144444, 11151119, 11151199, 11151519, 11151999, 11153519, 11155119, 11155199, 11155519, 11155555, 11166666, 11177777, 11188888, 11199999, 11222222, 11315119, 11315199, 11315519, 11315555, 11333333, 11351119, 11351199, 11351519, 11351999, 11411777, 11417777, 11441777, 11444444, 11453519, 11511199, 11511999, 11515199, 11519999, 11535199, 11551199, 11551999, 11555199, 11555555, 11666666, 11777777, 11888888, 11999999, 12222222, 13151119, 13151199, 13151519, 13151999, 13153519, 13155119, 13155199, 13155519, 13155555, 13333333, 13511199, 13511999, 13515199, 13519999, 14117777, 14177777, 14417777, 14444444, 14453519, 14535199, 15111999, 15119999, 15151999, 15199999, 15351999, 15511999, 15519999, 15551999, 15555555, 16666666, 17777777, 18888888, 19999999}

Si l'on ne s'occupe que de la liste de tous les chiffres obtenus, on voit qu'on a une suite quasi fractale, certains nombres ont une trajectoire évidente avant de se stabiliser :

4
14
144
1444
etc.

Pour d'autres c'est un peu moins évident :

9
19
519
3519
53519
453519
4453519
14453519
114453519
1114453519
11114453519
etc.

Vraiment étrange !

Entre 10^j et 10^(j+1) , on garde les mêmes nombres en rajoutant un 1 devant, mais on a aussi de nouvelles branches qui se créent, par ex. 151999 donne naissance à 1151999, mais aussi à 5151999

ou encore, quand on examine dans la suite les nombres ne contenant que des 1, des 4 et des 7 (par exemple) :

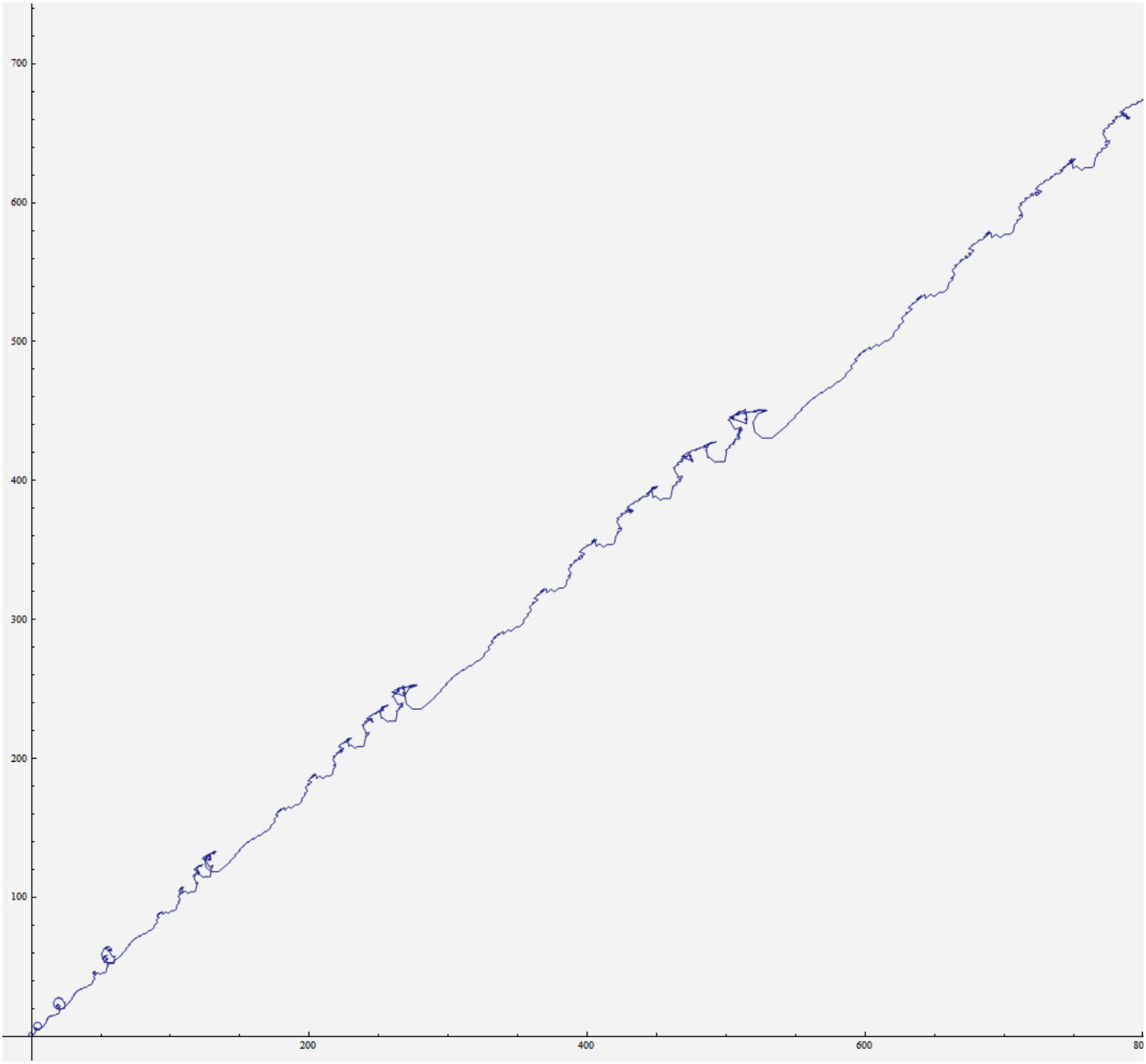
417
1417
4117
4177
4417
11417
14117
14177
14417
41177
41777
44177
111417
114117
114177
114417
141177
141777
144177
411777
417777
441777
1111417
1114117
1114177
1114417
1141177
1141777
1144177
1411777
1417777
1441777
4117777
4177777
4417777
11111417
11114117
11114177
11114417
11141177
11141777
11144177
11411777
11417777
11441777
14117777
14177777
14417777
...

on voit que les branches se développent régulièrement, mais créent aussi des rejetons !

Pour visualiser le côté "fractal perturbé" de cette suite, j'ai dessiné la trajectoire correspondant aux chiffres composant la suite des nombres, c'est à dire :

1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 1, 1, 2, 1, 3, 1, 4, 1, 5, 1, 6, 1, 7, 1, 8, 1, 9, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6, 7, 7, 8, 8, 9, 9, 1, 1, 1, 1, 1, 2, 1, 1, 3, 1, 1, 4, 1, 1, 5, 1, 1, 6, 1, 1, 7, 1, 1, 8, 1, 1, 9, 1, 2, 2, 1, 3, 3, 1, 4, 4, 1, 5, 5, 1, 6, 6, 1, 7, 7, 1, 8, 8, 1, 9, 9, 2, 2, 2, 3, 1, 5, 3, 3, 3, 4, 1, 7, 4, 4, 4, 5, 1, 9, 5, 5, 5, 6, 6, 6, 7, 7, 7, 8, 8, 8, 9, 9, 9, 1, 1, 1, 1, 1, 1, 1, 2, 1, 1, 1, 3, 1, 1, 1, 4, 1, 1, 1, 5, 1, 1, 1, 6...

en assignant une direction à chaque chiffre (1=40°, 2=80°, etc.) On voit que cette suite forme un motif qui se répète en se déformant peu à peu :



Magnifique, Jean-Marc, on voit bien ci-dessus le petit train de vagues qui monte et grossit !
Thanks to all contributors,
Best,
É.
P.-S. this is now http://oeis.org/A173646