Wolfdieter Lang, Jul 13 2007

A130561 tabf array: partition numbers, called by W.L. M_8 numbers; also M31(2) (in the paper "Combinatorial interpretation of generalized Stirling numbers", Oct 2008).

Partitions of n listed in Abramowitz-Stegun order p. 831-2 (see the main page for an A-number with the reference).

n\k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22 ...
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	6	6	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	24	24	12	12	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	120	120	120	60	60	20	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	720	720	720	360	360	720	120	120	180	30	1	0	0	0	0	0	0	0	0	0	0	0
7	5040	5040	5040	5040	2520	5040	2520	2520	840	2520	840	210	420	42	1	0	0	0	0	0	0	0
8	40320	40320	40320	40320	20160	20160	40320	40320	20160	20160	6720	20160	10080	20160	1680	1680	6720	3360	336	840	56	1
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.																						
n\k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22 ...

The next two rows, for n=9 and 10 are:

n=9: [[362880], [362880, 362880, 362880, 362880], [181440, 362880, 362880, 181440, 181440, 362880, 60480], [60480, 181440, 181440, 181440, 181440, 60480], [15120, 60480, 30240, 90720, 15120], [3024, 15120, 10080], [504, 1512], [72], [1]]

n=10: [[3628800], [3628800, 3628800, 3628800, 3628800, 1814400], [1814400, 3628800, 3628800, 3628800, 1814400, 3628800, 1814400, 1814400], [604800, 1814400, 1814400, 907200, 1814400, 3628800, 604800, 604800, 907200], [151200, 604800, 604800, 907200, 907200, 604800, 30240], [30240, 151200, 75600, 302400, 75600], [5040, 30240, 25200], [720, 2520], [90], [1]]]

The row sums give A000262(n) = [1, 3, 13, 73, 501, 4051, 37633, 394353, 4596553, 58941091,...]

This is also the sequence of row sums of the unsigned Lah triangle A105278.

###

Combinatorial interpretation:

a(n,k) counts the number of lists (ordered subsets) which partition the set {1,2,...,n}, with the lengths of the lists given by the k-th partition of n in A-St order.

E.g. a(5,5): lengths of the lists from the partition [1,2^2] (5-th partition of 5 in A-St order).

List structure [.],[..],[..] Hence, a(5,5)=binomial(5,2)*binomial(3,2)= 5!/(1!*2!) = 60 from distributing the numbers 1,2,...,5 over these lists.

E.g. a(4,3), list structure from partition [2^2]: [..],[..]. There are a(4,3)= 2*binomial(4,2)= 4!/2! = 12 possibilities to put the numbers 1,2,3,4 into these two lists, namely

{[1,2],[3,4]}, {[1,2],[4,3]} , {[2,1],[3,4]}, {[2,1],[4,3]},

{[1,3],[2,4]}, {[1,3],[4,2]} , {[3,1],[2,4]}, {[3,1],[4,2]},

{[1,4],[2,3]}, {[1,4],[3,2]} , {[4,1],[2,3]}, {[4,1],[3,2]}.

Note that, e.g., {[1,4],[2,3]} = {[2,3],[1,4]}. No order within lists of the same length.

##

The row polynomials are (in A-St order, as lists of monomials):

n=1: [x[1]]

n=2: [2*x[2], x[1]^2]

n=3: [6*x[3], 6*x[1]*x[2], x[1]^3]

n=4: [24*x[4], 24*x[1]*x[3], 12*x[2]^2, 12*x[1]^2*x[2], x[1]^4]

n=5: [120*x[5], 120*x[1]*x[4], 120*x[2]*x[3], 60*x[1]^2*x[3], 60*x[1]*x[2]^2, 20*x[1]^3*x[2], x[1]^5],

n=6: [720*x[6], 720*x[1]*x[5], 720*x[2]*x[4], 360*x[3]^2, 360*x[1]^2*x[4], 720*x[1]*x[2]*x[3], 120*x[2]^3,
120*x[1]^3*x[3], 180*x[1]^2*x[2]^2, 30*x[1]^4*x[2], x[1]^6]

n=7: [5040*x[7], 5040*x[1]*x[6], 5040*x[2]*x[5], 5040*x[3]*x[4], 2520*x[1]^2*x[5], 5040*x[1]*x[2]*x[4],
2520*x[1]*x[3]^2, 2520*x[2]^2*x[3], 840*x[1]^3*x[4], 2520*x[1]^2*x[2]*x[3], 840*x[1]*x[2]^3, 210*x[1]^4*x[3],
420*x[1]^3*x[2]^2, 42*x[1]^5*x[2], x[1]^7]

n=8: [40320*x[8], 40320*x[1]*x[7], 40320*x[2]*x[6], 40320*x[3]*x[5], 20160*x[4]^2, 20160*x[1]^2*x[6],
40320*x[1]*x[2]*x[5], 40320*x[1]*x[3]*x[4], 20160*x[2]^2*x[4], 20160*x[2]*x[3]^2, 6720*x[1]^3*x[5],
20160*x[1]^2*x[2]*x[4], 10080*x[1]^2*x[3]^2, 20160*x[1]*x[2]^2*x[3], 1680*x[2]^4, 1680*x[1]^4*x[4],
6720*x[1]^3*x[2]*x[3], 3360*x[1]^2*x[2]^3, 336*x[1]^5*x[3], 840*x[1]^4*x[2]^2, 56*x[1]^6*x[2], x[1]^8],

n=9: [362880*x[9], 362880*x[1]*x[8], 362880*x[2]*x[7], 362880*x[3]*x[6], 362880*x[4]*x[5], 181440*x[1]^2*x[7],
362880*x[1]*x[2]*x[6], 362880*x[1]*x[3]*x[5], 181440*x[1]*x[4]^2, 181440*x[2]^2*x[5], 362880*x[2]*x[3]*x[4],
60480*x[3]^3, 60480*x[1]^3*x[6], 181440*x[1]^2*x[2]*x[5], 181440*x[1]^2*x[3]*x[4], 181440*x[1]*x[2]^2*x[4],
181440*x[1]*x[2]*x[3]^2, 60480*x[2]^3*x[3], 15120*x[1]^4*x[5], 60480*x[1]^3*x[2]*x[4], 30240*x[1]^3*x[3]^2,
90720*x[1]^2*x[2]^2*x[3], 15120*x[1]*x[2]^4, 3024*x[1]^5*x[4], 15120*x[1]^4*x[2]*x[3], 10080*x[1]^3*x[2]^3,
504*x[1]^6*x[3], 1512*x[1]^5*x[2]^2, 72*x[1]^7*x[2], x[1]^9]

n=10: [3628800*x[10], 3628800*x[1]*x[9], 3628800*x[2]*x[8], 3628800*x[3]*x[7], 3628800*x[4]*x[6], 1814400*x[5]^2,
1814400*x[1]^2*x[8], 3628800*x[1]*x[2]*x[7], 3628800*x[1]*x[3]*x[6], 3628800*x[1]*x[4]*x[5], 1814400*x[2]^2*x[6],
3628800*x[2]*x[3]*x[5], 1814400*x[2]*x[4]^2, 1814400*x[3]^2*x[4], 604800*x[1]^3*x[7], 1814400*x[1]^2*x[2]*x[6],
1814400*x[1]^2*x[3]*x[5], 907200*x[1]^2*x[4]^2, 1814400*x[1]*x[2]^2*x[5], 3628800*x[1]*x[2]*x[3]*x[4],
604800*x[1]*x[3]^3, 604800*x[2]^3*x[4], 907200*x[2]^2*x[3]^2, 151200*x[1]^4*x[6], 604800*x[1]^3*x[2]*x[5],
604800*x[1]^3*x[3]*x[4], 907200*x[1]^2*x[2]^2*x[4], 907200*x[1]^2*x[2]*x[3]^2, 604800*x[1]*x[2]^3*x[3],
30240*x[2]^5, 30240*x[1]^5*x[5], 151200*x[1]^4*x[2]*x[4], 75600*x[1]^4*x[3]^2, 302400*x[1]^3*x[2]^2*x[3],
75600*x[1]^2*x[2]^4, 5040*x[1]^6*x[4], 30240*x[1]^5*x[2]*x[3], 25200*x[1]^4*x[2]^3, 720*x[1]^7*x[3],
2520*x[1]^6*x[2]^2, 90*x[1]^8*x[2], x[1]^10]]

e.o.f.