A120080/A120081, Wolfdieter Lang (revised July 16 2013)

Rationals A120080(n)/A120081(n) (expansion of original Debye function D(x) for |x| <2*pi). Case n=3 of the general D(n,x).

See the Landau, Lifschitz and Abramowitz-Stegun references.

r(n):=[x^n](1-3*x/8+sum((B(2*k)/((2*k+3)*(2*k)!))*x^(2*k),0,..infinity)) (in lowest terms), |x|<2*pi. B(2*k) are Bernoulli numbers.

For n=0..40 one has for r(n):

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 [1, -3/8, 1/20, 0, -1/1680, 0, 1/90720, 0, -1/4435200, 0, 1/207567360, 0,
-691/6538371840000, 0, 1/423437414400, 0, -3617/6758061133824000, 0,
43867/35763659520196608000, 0, -174611/6155242080686899200000, 0,
77683/1175091669949317120000000, 0, -236364091/15244417230585693025075200000, 0,
657931/17993003650263943741440000000, 0,
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1723168255201/8357693208106440751936262111232000000, 0,
-7709321041217/1565628479755476504494947172352000000000, 0,
151628697551/128513100758415920133669277138944000000000, 0,
-26315271553053477373/9281036346944836728181328191655818297344000000000, 0,
154210205991661/2257255506961120587595820651379267993600000000, 0,
-261082718496449122051/1582304508802648076216443935127747553761689600000000000]
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The original Debye function (see the Landau-Lifschitz reference) is defined as an integral function:

D(x):= (3/x^3)*int((t^3)/(exp(t)-1),t,0..x).

This is the n=3 instance of the generalized Debye function

D(n,x):= (n/x^n)*int((t^n)/(exp(t)-1),t,0..x).

See for D(0,x) =1; and for n=1: A120082(n)/A120083(n); n=2: A120084(n)/A120085(n), n=4: A120086(n)/A120087(n).

Note added (July 16 2013):
g(n,t) := t^n/(exp(t) - 1) is the e.g.f. of {risefac(k-(n-2),n-1)*B(k-(n-1))}. k >= n-1, and 0 for k=0..n-2. Here risefac(x,n) = x*(x+1)*...*(x+(n-1)) is the rising factorial. That is, diff(g(n,x),x$(n-1)) is the e.g.f. of the sequence {risefac(k+1,n-1)*B(k)}, k >= 0, n = 1, 2, ...
Using the e.g.f. g(n,t) and term by term integration (allowed for |x| <= r < rho, with some small enough rho) one finds that D(n,x) is the e.g.f. of the sequence {n*B(k)/(k+n)}, k >= 0, n=1, 2, 3,... Therefore, D(n,x) is the o.g.f. of the sequence {n*B(k)/((k+n)*k!)}, k >= 0.
This gives for n=3 another formula for the rationals r(n) = {3*B(n)/((n+3)*n!)}, and the sequences A120080(n) = numerator(r(n)) and A120081(n) = denominator(r(n)) (in lowest terms).

############################ e.o.f. ##