

Wolfdieter Lang, Oct 04 2012

$$x^2 - x*y - G*y^2 = k \text{ with } k = +1 \text{ or } -1$$

Take minimal (positive) solution  $(x(n), y(n))$  of these two Diophantine equations.

$$x(n) = A077057(n), \quad y(n) = A077058(n) \text{ with } G(n) = A078358(n).$$

The  $k = +1$  (abbreviated by +) equation has always solutions (see the Perron, Vol. I reference, Satz 3.35, p. 109), which will also be given below. If also the  $k = -1$  (abbreviated by -) equation has a solution then the smallest of both solutions is taken, indicated by an  $m$  below the solution. In the Perron table, p. 108, brackets in the third column appear whenever there is no  $k = -1$  solution (indicated below by \*), namely for  $G = 5, 8, 11, 14, 17, 19, 23, 26, 29, 32, 33, \dots$  OEIS A217470.

n	1	2	3	4	5	6	7	8	9	10	11	12
G(n)	1	3	4	5	7	8	9	10	11	13	14	15
k	+ -	+ -	+ -	+ -	+ -	+ -	+ -	+ -	+ -	+ -	+ -	+ -
x(n)	2 1	7 2	41 5	3 *	16 3	27 *	85 7	2363 37	4 *	29 4	171 *	859 22
y(n)	1 1	3 1	16 2	1 *	5 1	8 *	24 2	640 10	1 *	7 1	40 *	195 5
		m	m		m		m	m		m		m

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n	13	14	15	16	17	18	19	20
G(n)	16	17	18	19	21	22	23	24
k	+ -	+ -	+	-	+ -	+	-	+
x(n)	145 9	14 *	2548249	1193	5 *	46 5	553001	553
y(n)	32 2	3 *	53400	250	1 *	9 1	106000	106
		m		m		m	m	

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n	21	22	23	24	25	26	27	28
G(n)	25	26	27	28	29	31	32	33
k	+ -	+ -	+	-	+ -	+ -	+	-
x(n)	221 11	45 *	37324	143	1317649	849	6 *	67 6
y(n)	40 2	8 *	6525	25	226592	146	1 *	11 1
		m		m		m	m	

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n	29	30	31	32	33	34	35	36
G(n)	34	35	36	37	38	39	40	41
k	+	-	+ -	+ -	+ -	+ -	+ -	+ -
x(n)	6602785	1893	103 *	313 13	2014 33	2353 *	24496	115
y(n)	1039424	298	16 *	48 2	305 5	352 *	3621	17
		m		m	m		m	

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n	37	38	39	40	41	42	43		
G(n)	43	44	45	46	47	48	49		
k	+ -	+ -	+ -	+ -	+ -	+ -	+ -		
x(n)	92 7	67115 *	914806	701	9929 73	59 *	6672360030889	1891117	421 15
y(n)	13 1	9384 *	126585	97	1360 10	8 *	896073208080	253970	56 2
	x		x	x		x	x	x	

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n	44	45	46	47	48	49	50	51	
G(n)	50	51	52	53	54	55	57	58	
k	+ -	+ -	+ -	+ -	+ -	+ -	+ -	+ -	
x(n)	551427 *	23 *	49771 *	39 *	4105015 *	8 *	121 8	1142655977	24673
y(n)	72664 *	3 *	6440 *	5 *	521904 *	1 *	15 1	140510608	3034
							x		x

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n	52	53	54	55	56	57	
G(n)	59	60	61	62	63	64	
k	+ -	+ -	+ -	+ -	+ -	+ -	
x(n)	41 *	10734784762045849	75585293	25 *	9095891 *	989 *	545 17
y(n)	5 *	1299282410089200	9148450	3 *	1084152 *	117 *	64 2
			x				x

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n	58	59	60	61	62	63			
G(n)	65	66	67	68	69	70			
k	+ -	+ -	+ -	+ -	+ -	+ -			
x(n)	386 *	78268081	6445	14269 87	771 *	3619006	1385	2397152205529	1126977
y(n)	45 *	9059424	746	1640 10	88 *	410241	157	269903150960	126890
		x	x		x				x

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