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Russian

Emails

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 fqk 7829

From math.gatech.edu!ebussian Thu Oct 13 15:42:38 EDT 1994
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Date: Thu, 13 Oct 94 15:42:38 EDT
From: ebussian@math.gatech.edu (Eric Bussian 1/93-//93)
Message-Id: <9410131942.AA12851@math>
To: njas@research.att.com
Subject: Re: Your inquiry about a sequence I sent
Cc: ebussian@math.gatech.edu
Status: R

Dr. Sloane,

You asked for a description of the sequence I sent to your sequence checker. Professor Richard Duke and I have spent the last few months studying random walks on graphs and questions regarding the time to cover all the directed edges of a graph during a random walk. The sequence in which you expressed interest was generated while we were trying to determine the expected number of steps necessary to cover the edges of K3. We developed a system of linear recurrence relations which generated the sequence but ended up solving the larger problem from a different perspective and did not pursue a closed form description of the sequence. A more complete description of the sequence is attached at the end of this message.

We've come across an even more interesting sequence {b(n)} while studying the edge cover time for the directed version of the path on n vertices. However, it's not an integer sequence.

The sequence satisfies the following:

Sum 1 <= k <= (n-1) of (1/(n-k)) * b(k) = 1

Which in turn leads to:

Sum 1 <= k <= (n-1) of H(n-k) * b(k) = (n-1),

where H(j) is the jth Harmonic number.

The generating function for the b(n)s is:

(x^2)/(1-x) * Log(1/(1-x))

Wilf notes that c(j) = b(j) * (j+2)! is an integer for j <= 20.
He sent the first 20 c(j) to your sequence checker but received a null response.

Eric Bussian
ebussian@math.gatech.edu

--------------------- Sequence Description --------------------

This sequence comes up in the study of a random walk on the directed multigraph induced by K3, i.e. a directed graph on three vertices in which there is a directed edge connecting every ordered pair of vertices. The random walk we consider may be viewed as placing a token on vertex #1 and at each step in time allowing it to travel along the directed edges of the graph, choosing an edge with uniform probability from those outedges incident to its present position. Since each vertex has outdegree = 2, after n steps there are 2^n sequences of vertices recording the travel of the token through the graph. A(n) counts exactly those sequences that on step n have for the first time traversed all six directed edges in the graph, i.e. A(n) counts those sequences which on step n-1 have traversed exactly five of the six edges and which on step n traverse the remaining edge. The {A(n)} were of interest while we were attempting to determine the expected number of steps necessary for a random walk to have traversed all edges at least once.

The sequence may be generated by a system of linear recurrence relations which describe the fourteen possible configurations of the walk. At this time we've answered our question regarding the edge cover time by other means and have not pursued a closed form description of the sequence.

0,0,0,0,6,8,28,44,100,162,318,514,942,1518,2672,4302,7380,11882,
20040,32276,53810,86710,143396,231204,380152,613286,1004188,
1620864,2645928,4272744