SCAN A7380 etc

Yanco

~~Letter~~ + emails

5 pages

Many segs

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Dear Dr. Sloane,

I was very pleased to learn that your *Handbook of Integer Sequences* is now available through the Internet. Your book, twenty years old, has proven invaluable to me; and a frequently-updated list of sequences is even better.

I am enclosing two things: one, a preprint of an article submitted by Ansuman Bagchi and myself to *J. Graph Theory*; two, some charts of numbers to which the article refers (but which it does not explicitly provide).

In my opinion there are four classes of sequences inherent to the charts; let me give each in turn.

A–I 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, ...

A–II 1, 2, 2, 3, 4, 5, 7, 9, 12, 16, 21, 28, 37, 49, 65, 86, 114, 151, 200, 265, 351, 465, 616, 816, 1081, 1432, 1897, 2513, 3329, ...

A–III 1, 2, 1, 3, 2, 4, 4, 5, 7, 7, 11, 11, 16, 18, 23, 29, 34, 45, 52, 68, 81, 102, 126, 154, 194, 235, 296, 361, 450, 555, 685, 851, 1046, ...

A–IV 1, 2, 1, 3, 1, 4, 2, 5, 4, 6, 7, 7, 11, 9, 16, 13, 22, 20, 29, 31, 38, 47, 51, 69, 71, 98, 102, 136, 149, 187, 218, 258, 316, 360, 452, ...

A–V 1, 2, 1, 3, 1, 4, 1, 5, 2, 6, 4, 7, 7, 8, 11, 9, 16, 11, 22, 15, 29, 22, 37, 33, 46, 49, 57, 71, 72, 100, 94, 137, 127, 183, 176, 240, ...

These sequences correspond to the number of $k$-th order maximal independent sets (as defined in the aforementioned article) in a path graph, for $k = -1$, 1, 3, 5, and 7, respectively. For $k = -1$, we see the Fibonacci sequence [256]. For $k \geq 1$, we discover an infinite family of "Fibonacci sequences" first located by Green and cited in your book: $k = 1$ is precisely sequence 102; $k = 3$ is sequence 59.

If you look at the first five charts (corresponding to path graphs), the rightmost nonzero entry in each row corresponds to the number of *maximum* independent sets

in a path graph of that size; these numbers by definition are on the chart for all $k$. Hence if we subtract 1, 2, 1, 3, 1, 4, 1, 5, ...from each of the above sequences, we locate our second class of sequences. Bagchi calls these the number of *strict* $k$-th order maximal independent sets; "strict" because they are maximal but not maximum. (Additionally, for $k = -1$ I am disregarding the column of ones.)

B–I 0, 0, 3, 4, 11, 16, 32, 49, 87, 137, 231, 369, 608, 978, 1595, 2574, ...

B–II 0, 0, 1, 0, 3, 1, 6, 4, 11, 10, 20, 21, 36, 41, 64, 77, 113, 141, 199, 254, 350, 453, 615, 803, 1080, 1418, 1896, 2498, 3328, ...

B–III 0, 0, 0, 0, 1, 0, 3, 0, 6, 1, 10, 4, 15, 10, 22, 20, 33, 35, 51, 57, 80, 90, 125, 141, 193, 221, 295, 346, 449, 439, 684, 834, 1045, 1283, ...

B–IV 0, 0, 0, 0, 0, 0, 1, 0, 3, 0, 6, 0, 10, 1, 15, 4, 21, 10, 28, 20, 37, 35, 50, 56, 70, 84, 101, 121, 148, 171, 215, 241, 315, 342, 451, ...

B–V 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 3, 0, 6, 0, 10, 0, 15, 1, 21, 4, 28, 10, 36, 20, 45, 35, 56, 56, 71, 84, 93, 120, 126, 165, 175, 221, ...

(To the best of my knowledge, none of those sequences are in your list.)

We can do the same for cycle graphs. Here are the sequences corresponding to the number of $k$-th order maximal independent sets in *cycle* graphs, for $k = -1$, 1, 3, 5, and 7:

C–I 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, ...

C–II 0, 2, 3, 2, 5, 5, 7, 10, 12, 17, 22, 29, 39, 51, 68, 90, 119, 158, 209, 277, 367, 486, 644, 853, 1130, 1497, 1983, 2627, 3480, 4610, 6107, 8090, ...

C–III 0, 2, 3, 2, 5, 2, 7, 2, 9, 7, 11, 14, 13, 23, 20, 34, 34, 47, 57, 67, 91, 101, 138, 158, 205, 247, 306, 387, 464, 592, 713, 898, 1100, 1362, 1692, ...

C–IV 0, 2, 3, 2, 5, 2, 7, 2, 9, 2, 11, 2, 13, 9, 15, 18, 17, 29, 19, 42, 28, 57, 46, 74, 75, 93, 117, 121, 174, 167, 248, 242, 341, 359, 462, 533, ...

C–V 0, 2, 3, 2, 5, 2, 7, 2, 9, 2, 11, 2, 13, 2, 15, 2, 17, 11, 19, 22, 21, 35, 23, 50, 25, 67, 36, 86, 58, 107, 93, 130, 143, 155, 210, 191, ...

For $k = -1$, we see the Lucas sequence [924]. For $k = 1$, we see the Perrin sequence [163] (later located by D. Fielder). For $k \geq 3$, we discover an infinite family of "Perrin sequences" — made interesting because $p|A_k(p)$ for all prime $p$ (e.g., the 29th element of each sequence is divisible by 29.)

For cycle graphs, the number of maximum independent sets follows the sequence 0, 2, 3, 2, 5, 2, 7, 2, 9, 2, 11, ..., so we can subtract those values from the sequence. (For $k = -1$ I am again disregarding the column of ones.)

D–I 0, 0, 0, 4, 5, 15, 21, 44, 66, 120, 187, 320, 508, 841, 1349, 2205, ...

D–II 0, 0, 0, 0, 0, 3, 0, 8, 3, 15, 11, 27, 26, 49, 53, 88, 102, 156, 190, 275, 346, 484, 621, 851, 1105, 1495, 1956, 2625, 3451, 4608, 6076, 8088, ...

D–III 0, 0, 0, 0, 0, 0, 0, 0, 0, 5, 0, 12, 0, 21, 5, 32, 17, 45, 38, 65, 70, 99, 115, 156, 180, 245, 279, 385, 435, 590, 682, 896, 1067, 1360, ...

D–IV 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 7, 0, 16, 0, 27, 0, 40, 7, 55, 23, 72, 50, 91, 90, 119, 145, 165, 217, 240, 308, 357, 427, 531, ...

D–V 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 9, 0, 20, 0, 33, 0, 48, 0, 65, 9, 84, 29, 105, 62, 128, 110, 153, 175, 189, ...

See email

If these sequences seem interesting enough for inclusion in the online *Handbook* let me know — I will confirm the numbers and send them to you. (The charts were thrown together in a few hours for a presentation I was giving, and I did quite a bit of calculating in my head. Consequently there may be errors.)

Additionally some interesting sequences exist for the number of "equivalence classes" of $k$-th order maximal independent sets in paths and cycles. I could provide you with those as well.

Yours truly,

Richard J. Vanco

From unix.amherst.edu!rjyanco Sun May 1 23:37:22 0400 1994
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From: "Richard J. Yanco" <rjyanco@unix.amherst.edu>
Message-Id: <199405020337.AA10376@amhux3.amherst.edu>
Subject: confirmed sequences
To: njas@research.att.com
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Cc: rjyanco@unix.amherst.edu (Richard J. Yanco)
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Greetings:

I received your letter Friday and was able to verify the sequences today. Presumably you have the original letter, so I will simply label the sequences as they were labelled there.

A-I: [256]

A-II: [102]

A-III:[59]

A-IV: 1 2 1 3 1 4 2 5 4 6 7 7 11 9 16 13 22 20 29 31 38 47 51 69 71 98 102 136 149 187 218 258 316 360 452 509 639 727 897 1043

A-V: 1 2 1 3 1 4 1 5 2 6 4 7 7 8 11 9 16 11 22 15 29 31 37 33 46 49 57 71 72 100 94 137 127 183 176 240 247 312 347 406

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B-I: 0 0 3 4 11 16 32 49 87 137 231 369 608 978 1595 2574 4179 6754 10944 17699 28655 46355 75023 121379 196416 317796 514227 832024 1346267

B-II: 0 0 1 0 3 1 6 4 11 10 20 21 36 41 64 77 113 141 199 254 350 453 615 803 1080 1418 1896 2498 3328 4394 5841 7722 10251 13563 17990 23814 31571 41804 55404 73375

B-III:0 0 0 0 1 0 3 0 6 1 10 4 15 10 22 20 33 35 51 57 80 90 125 141 193 221 295 346 449 539 684 834 1045 1283 1600 1967 2451 3012 3752 4612 5738 7063

B-IV: 0 0 0 0 0 0 1 0 3 0 6 0 10 1 15 4 21 10 28 20 37 35 50 56 70 84 101 121 148 171 217 241 315 342 451 490 638 707 896 1022 1256 1473 1765 2111

B-V: 0 0 0 0 0 0 0 0 1 0 3 0 6 0 10 0 15 1 21 4 28 10 36 20 45 35 56 56 71 84 93 120 126 165 175 221 246 292 346 385 483 511 666 686 906 932

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C-I: [924]

C-II: [163]

7387

C-III:0 2 3 2 5 2 7 2 9 7 11 14 13 23 20 34 34 47 57 67 91 101 138 158 205
279 306 387 464 592 713 898 1100 1362 1692 2075 2590 3175 3952 4867 6027
7457 9202 11409 14069 17436 21526 26638

7388

C-IV: 0 2 3 2 5 2 7 2 9 2 11 2 13 9 15 18 17 29 19 42 28 57 46 74 75 93 117
121 174 167 248 242 242 341 359 462 533 629 781 871 1122 1230 1584 1763 2213
2544 3084 3666 4314 5250 6077 7463 8621 10547 12287 14861

C-V: 0 2 3 2 5 2 7 2 9 2 11 2 13 2 15 2 17 11 19 22 21 35 23 50 25 67 36 86
58 107 93 130 143 155 210 191 296 249 403 342 533 485 688 695 879 991
1128 1394 1470 1927 1955 2615 2650 3494 3641 4622 5035 6092 6962 8047

7389

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7390

D-I: 0 0 0 4 5 15 21 44 66 120 187 319 507 840 1348 2204 3553 5776
9329 15124 24454 39600 64055 103679 167735 271440 439176 710644 1149821

D-II: 0 0 0 0 0 3 0 8 3 15 11 27 26 49 53 88 102 156 190 275 346 484 621 851
1105 1495 1956 2625 3451 4608 6076 8088 10684 14195 18772 24912 32967
43719 57879 76723 101598 134641 178321 236280

7391

D-III:0 0 0 0 0 0 0 0 0 5 0 12 0 21 5 32 17 45 38 65 70 99 115 156 180 247
279 385 435 590 682 896 1067 1360 1657 2073 2553 3173 3913 4865 5986 7455
9159 11407 14024 17434 21479

7392

D-IV: 0 0 0 0 0 0 0 0 0 0 0 0 0 0 7 0 16 0 27 0 40 7 55 23 72 50 91 90 119
145 165 217 240 308 357 427 531 592 779 832 1120 1189 1582 1720 2211 2499
3082 3619 4312 5201 6075 7412

7393

V: 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 9 0 20 0 33 0 48 0 65 9 84 29 105
62 128 110 153 175 189 259 247 364 340 492 483 645 693 834 989 1081 1392 1421
1925 1904 2613 2597 3492 3586

7394

Barring typographical errors these numbers are correct; I checked them twice.

Regards,

Richard Yanco
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