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A Note on the Distribution of Primes in Arithmetic Progressions

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There is an exercise in several texts on number theory that sheds light on the distribution of prime numbers in the sequence $[n]$ of positive integers. This exercise may be stated as follows: find a sequence of k consecutive members of $[n]$ all of which are composite. Do this for any $k > 1$. The solution (most often hinted at) is to take $N = (k + 1)! + 1$ and consider the numbers $N + 1, N + 2, \ldots, N + k$. These are all composite being divisible by $2, 3, \ldots, k + 1$ respectively. Both the problem and its solution are easy to grasp. Moreover they spotlight how rare indeed is the occurrence of prime numbers in the sequence $[n]$ as opposed to that of nonprimes. We will point out in a similar fashion that the primes in an arithmetic progression $[an + b]$, $(a,b) = 1$, have a like distribution.

Given $[an + b]$ with $(a,b) = 1$, the progression does contain infinitely many primes. This is the celebrated theorem of Dirichlet. Suppose we are asked to find k consecutive members of $[an + b]$, all composite. Set $N = \prod_1^k (an + b)$ and consider the members $aN + a + b, aN + 2a + b, \ldots, aN + ka + b$. This string of k numbers from the progression are respectively divisible by $a + b, 2a + b, \ldots, ka + b$. None of these divisors is 1 and none equals its respective dividend.

For an example, given $[2n + 5]$ we are asked for a string of 7 composite numbers. Here $k = 7$, $N = 43648605$ and the successive non-primes are 87297217, 87297219, 87297221, 87297223, 87297225, 87297227, 87297229 which are divisible respectively by 7, 9, 11, 13, 15, 17, and 19.

Consecutive-Digit Primes–Again

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In an earlier issue of this journal [5] the facts then known about consecutive-digit primes were given. A consecutive-digit prime is one formed of digits in either ascending or descending order. It is shown that there are no primes starting with 9 and using digits in descending order; that only one prime was then known which began with 1 and used

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6055

consecutive digits in ascending order (namely, 1234567891); and that there were a few primes with digits in consecutive order, not starting with 1 or 9.

The problem was then posed to seek other consecutive-digit primes beginning with 1 and with digits in ascending order. However, several readers submitted results of studies involving consecutive digits in ascending and descending order and starting with any digit.

Don Eastlake [2], Raphael Finkelstein [3], Henry Greenwald and John F. Barrett [4], Rich Schroeppel and the M.I.T. Mathlab Group [8], and the author [5, 6, 7] devoted considerable time to searching for these primes up to 21 or 22 digits in length. The table following (with references noted) is a complete listing of such primes known which have less than 22 digits. The surprise is the paucity of descending-order consecutive-digit primes. Complete factorizations for most of the consecutive-digit composite integers are available. A program which could handle integers larger than 21 digits would probably dispose of many potential candidates. However, the occurrence of a prime with 22 or more digits could tie up even a fast computer for a long time! Further research should be approached with caution and unlimited computer time.

Digits in Ascending Order		Digits in Descending Order	
23	[1]	43	[1]
67	[1]	109	[1]
89	[1]	1987	[6]
4567	[1]	10987	[1]
~~67891~~	[1]	76543	[1]
78901	[1]		
~~89123~~	[6]		
678901	[1]		
~~4567891~~	[6]		
23456789	[1]		
45678901	[1]		
~~56789123~~	[6]		
~~1234567891~~	[5]		
~~4567891234567~~	[7]		
9012345678901	[7]		
789012345678901	[7]		
~~890123456789123~~	[7]		
~~1234567890123456789~~	[2]		

TABLE 1. Primes With Consecutive Digits

References

1. L. E. Card, "Patterns in Primes–Addenda", *JRM*, 1, 4, October 1968, pp. 250-252.
2. Don Eastlake, Artificial Intelligence Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts. (Factorizations and a prime received from Martin Gardner April 5, 1971.)
3. Raphael Finkelstein, Department of Mathematics, Bowling Green State University, Bowling Green, Ohio. (Factorizations received October 31, 1971 and April 6, 1972.)
4. Henry Greenwald and John F. Barrett, Chicago, Illinois. (Factorizations received March 16, 1972 and March 23, 1972.)
5. Joseph S. Madachy, "A Consecutive-Digit Prime", *JRM*, 4, 2, April 1971, page 100.
6. Joseph S. Madachy, searching through prime tables.
7. Joseph S. Madachy, found during April and May 1972 on an IBM 360/50.
8. Rich Schroeppel, who wrote the program, and the M.I.T. Mathlab Group for most of the computing time. (Factorizations of the descending-order consecutive-digit integers with 16 to 22 digits received from Michael Beeler May 15, 1972.)

6055

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February 12, 1974

Dr. Joseph S. Madachy
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Dear Dr. Madachy:

(1) In the January 1972 issue of JRM, N. Yoshigahara asks for a formula for the "card-return numbers" (on Page 37), but says that this may be a formidable problem. The answer is that the card-return number for 2n cards is the exponent of 2 mod 2n-1. In other words, it is the smallest positive integer e such that $2^e \equiv 1 \pmod{2n-1}$. E.g. when 2n=52, successive powers of 2 are 2, 4, 8, 16, 32, 64 = 13 (mod 51), 26, 52 = 1 (mod 51). Therefore e = 8, agreeing with page 37. Probably someone else has already pointed this out to you. But if not, let me know and I will write a short note about it for you.

(2) By the way, I found this answer by looking the sequence up in my Handbook of Integer Sequences, just published by Academic Press (see enclosure). I am also enclosing a list of interesting puzzle sequences that I made up some time ago for my friends. They are extracted from the book. Do you think it would be suitable for JRM?

(3) Finally, you have a very interesting note on "Consecutive-Digit Primes - Again", in Volume 5, October 1972, pp. 253-254. Do you happen to know how complete the list is that you give in Table I? In other words, let a_n denote the n-th prime number with consecutive ascending digits, so that $a_1 = 23$, $a_2 = 67$,... Then my question is this: how far has this sequence been rigorously computed? Any information you can send will be much appreciated.

Best regards,

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N. J. A. Sloane