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R VGuy
letter
3 pages
Neil J.A. Sloane,
AT&T Bell Laboratories, Room 2C-376,
600 Mountain Avenue,
Murray Hill, New Jersey 07974

Dear Neil,

This can serve as a cover for another letter I'm copying to you. Elsewhere in the Math. Gaz. issue that that refers to, is Note 71.37 on p.295, which took me to Math. Gaz. a year earlier, and thence to p.797 of Abramowitz & Stegun, Table 22.7 of coefficients in the Chebyshev polynomials $C_n(x)$, diagonals of which, ignoring signs, read:

1; $N$(Sloane #173); $N(N+3)/2$ (Sloane #522); and then, each being the differences of the one after, $N(N+1)...(N+k-2)(N+2k-1)/k!$, $k = 3,4,5,...$, i.e.


2,9,25,55,105,182,294,450,660,935,1287,1729,2275,2940,3740,4692,5814,7125,8645,10395,12397,14674,17250,20150,23400,...

2,11,36,91,196,378,672,1122,1782,2717,4004,5733,8008,10948,14688,19380,25194,32319,40964,51359,63756,78430,95680,115830,139230,...

2,13,49,140,336,714,1386,2508,4290,7007,11011,16744,24752,35700,50388,69768,94962,127821,168245,219604,...

Reading these another way, we get: 2; the odd numbers (not in Sloane?); the squares (S. #1350; square pyramids (S. #1774); 4-D (cubic?) pyramids (S. #1714, worth adding $N^2(N^2-1)/12$); and (presumably the 5-D tesseract pyramids) $N(N+1)(N+2)(N+3)(2N+3)/5!$

1,7,27,77,182,378,714,1254,2079,3289,5005,7371,10556,14756,20196,27132,35853,46683,59983,76153,95634,118910,...
Reading similarly in Table 22.5 (p. 796) of coefficients in
\( u_n(x) \), we get: powers of 2 (S. #432; S. #1396; S. #1729 (a very good
number, occurring in the second series of this letter); S. #1916; and
generally \( N(N+1)...(N+k-1) 2^{N-k}/k! \), which, for \( k = 4 \), is:

\[
1, 10, 60, 280, 1120, 4032, 13440, 42240, 126720, 366080, 1025024, 2795520,
7454720, 19496960, 50135040, 127008768, 317521920, ...
\]

Above the main diagonal of this table we get 1; S. #173;
S. #522; (as at the beginning of this letter) but these are
\( N(N+k+1)...(N+2k-1)/k! \) for \( k = 0,1,2 \), and now, for \( k = 3,4 \)
(the first member of the sequence is a Catalan number, so Sloane needs
to prefix a 1, though the real number (really!) to go in there is zero):

\[
5, 14, 28, 48, 75, 110, 154, 208, 273, 350, 440, 544, 663, 798, 950, 1120, 1309,
1518, 1748, 2000, 2275, 2574, 2898, ...
\]

\[
5, 87, 0, 14, 42, 90, 165, 275, 429, 637, 910, 1260, 1700, 2244, 2907, 3705, 4655, 5775,
7084, 8602, 10350, 12350, 14625, 17199, 20097, ...
\]

The corresponding sequences above the diagonal of Table 22.7
are 1; S. #173; S. #1002; S. #1363; S. #1578; S. #1719; S. #1847;
a familiar enough set of sequences, but not immediately recognizable
here, because they don't start at the beginning.

Above the diagonal of Table 22.8 is the same as above that
of 22.5.

No time now to comb through Tables 22.9, 22.10, 22.12 to
make sure you've done more justice to Legendre, Laguerre & Hermite
than you did to Chebyshev.

Best wishes,

Yours sincerely,

Richard K. Guy.

encl: qc of letter to S.N. Anderson
qc of Math. Gaz. article
\[ \frac{n(n+k+1) \cdots (n+2k+1)}{k!} \]

For \( k = 2 \), we have:

\[ \frac{n(n+3)}{2} \]

For \( k = 3 \), we have:

\[ \frac{n(n+4)(n+5)}{6} \]

For \( k = 4 \), we have:

\[ \frac{n(n+5)(n+6)(n+7)}{24} \]

For \( k = 5 \), we have:

\[ \frac{n(n+6)(n+7)(n+8)}{120} \]