

Scan A5315

J. A. Reeds,  
J. E. Knuth

NJ AB

3 pages  
emails

1 sequence

5315

From reeds Thu May 16 20:47:17 EDT 1991  
Status: R

a[1]=1  
a[2]=2

JA Reeds  
DE Knuth  
NJA's  
email

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a[3]=8
a[4]=42
a[5]=262
a[6]=1828
a[7]=13820
a[8]=110954
a[9]=933458
a[10]=8152860
a[11]=73424650
a[12]=678390116
a[13]=6405031050
a[14]=61606881612
a[15]=602188541928
a[16]=5969806669034

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>From gauss!arpa!SAIL.Stanford.EDU!DEK Sun Jan 29 00:06:50 1989
Message-ID: <4gbTS@SAIL.Stanford.EDU>
Date: 28 Jan 89 2107 PST
From: Don Knuth <DEK@SAIL.Stanford.EDU>
Subject: meander numbers
To: reeds%gauss@RESEARCH.ATT.COM, las@RESEARCH.ATT.COM
CC: VRP@SAIL.Stanford.EDU

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Vaughan Pratt couldn't resist computing those numbers on his SUN workstation, using a recurrence I thought of (an improvement of Koehler's approach)... The recurrence involves  $nC[n]$  values to compute the meanders that cross  $2n$  times, where  $C[n]$  is the  $n$ th Catalan number. (Koehler's approach had  $n^{2C[n]}$  for the strip-of-stamps problem.) Here are Vaughan's results, which agree with yours up to  $n=14$  (the largest value you sent me):

[the first experiments, Thursday morning 26 Jan]

n	A[n]	wall clock secs	max f(alpha,k)
11	73424650	8	4210
12	678390116	32	12198
13	6405031050	117	37378

[to go further meant going from 16 bits to 32, with paging onto disk; so he implemented a caching scheme, which slowed down the calculations:]

n	A[n]	wall clock secs	cache hits	cache accesses
6	1828	0	638	848
7	13820	1	2487	3279
8	110954	0	9658	12675
9	933458	1	37469	48915
10	8152860	6	148974	199360
11	73424650	20	595854	818026
12	678390116	85	2543840	3811940
13	6405031050	454	12649463	22757106
14	61606881612	2358	61337863	124032186
15	602188541928	12624	308321165	681140900

[then on Friday he had the value of A[16] but deleted it accidentally, so he has to compute it all again! We'll send A[16] soon. It looks like A[17] will be the limit of this particular approach; we need about  $4^n$  units of memory as well as time. Vaughan can save the memory when computing

A[17], because he won't have to store the values that would otherwise be used to make A[18]. I suppose the 50% hit rate in the cache can be improved somehow, but still the numbers need to be stored somewhere...]

>From gauss!arpa!SAIL.Stanford.EDU!DEK Mon Jan 30 23:00:32 1989  
Message-ID: <1\$hZH1@SAIL.Stanford.EDU>  
Date: 30 Jan 89 1149 PST  
From: Don Knuth <DEK@SAIL.Stanford.EDU>  
Subject: A16  
To: reeds%gauss@RESEARCH.ATT.COM, las@RESEARCH.ATT.COM

28-Jan-89 2221 coraki!pratt@Sun.COM A16  
Received: from Sun.COM by SAIL.Stanford.EDU with TCP; 28 Jan 89 22:21:26 PST  
Received: from sun.Sun.COM (sun-bb.sun.com) by Sun.COM (4.1/SMI-4.0)  
id AA05027; Sat, 28 Jan 89 22:22:47 PST  
Received: from coraki.UUCP by sun.Sun.COM (4.0/SMI-4.0)  
id AA06143; Sat, 28 Jan 89 22:20:36 PST  
Received: by (4.0/SMI-4.0Beta)  
id AA13341; Sat, 28 Jan 89 22:20:19 PST  
Date: Sat, 28 Jan 89 22:20:19 PST  
From: Vaughan Pratt <coraki!pratt@Sun.COM>  
Message-Id: <8901290620.AA13341@>  
To: dke@sail.stanford.edu  
Subject: A16  
Cc: coraki!pratt@Sun.COM

n	An	wall secs	cache hits	cache calls
13	6405031050	455	12649463	22757106
14	61606881612	2388	61337863	124032186
15	602188541928	13013	308321165	681140900
16	5969806669034	74844	1725643824	4147489672