Scan AS186
A259 881

R K Guy

S N Anderson

NJAS

Correspondence 1988

2 pages

2 Sep

T91

A5186
A259881

THE UNIVERSITY OF CALGARY

Faculty of SCIENCE
Department of MATHEMATICS & STATISTICS

2500 University Drive N.W., Calgary, Alberta, Canada T2N 1N4

Telephone (403) 220-5202

88-02-11

Stuart N. Anderson,
East Texas State University,
Commerce, TX 75428

Dear Stuart Anderson,

I was interested in your article, "Struggling with the $3x+1$ problem" *Math. Gaz.* 71 (1987) 271-274, as you might guess from the enclosed offprint. Particularly in your sequence

1,1,1,1,1,2,2,4,4,6,6,8,10,14,18,24,29,36,44,....

A5186
A259881

(where your 28 is presumably a misprint for 18) of numbers of integers x such that $f^n(x) = 1$, where $f(x)$ is the familiar $x/2$ or $3x+1$ function. Did you calculate this any further? I make the next few terms

.... 58,72,91,113,143, ...

and *estimate* the ones after that as

... 180?,227?,287?,361?,457?,576?,728?, ...

There are at least heuristic reasons for believing that an asymptotic estimate for the nth term is $A((3+\sqrt{21})/6)^n$ with $A \approx 0.65$. Perhaps this can be made rigorous, and the value of A determined?

I will copy this letter to Neil Sloane, because your sequence is not in his Handbook of Integer Sequences, and to Jeff Lagarias, because he keeps a bibliography of attempts on the $3x+1$ problem.

Yours sincerely,

Richard K. Guy.

RKG:1

Richard K. Guy.

encl: 136
φc N.J.A. Sloane ✓
J.C. Lagarias

THE UNIVERSITY OF CALGARY

Faculty of SCIENCE
Department of MATHEMATICS & STATISTICS

2500 University Drive N.W., Calgary, Alberta, Canada T2N 1N4

Telephone (403) 220-5202

88-08-18

Stuart N. Anderson,
East Texas State University,
Department of Mathematics,
307 Binnion Hall, East Texas Station,
Commerce, Texas 75428

Dear Stuart Anderson,

Thank you for your letter of 88-08-03, and the further calculations of the sequence that you originated in *Math. Gaz.* 71 (1987) 271-274. The numbers of integers x such that $f^n(x) = 1$, where $f(x)$ is the familiar $x/2$ or $3x+1$ function, for $n = 0,\ldots,$ namely

1,1,1,1,1,2,2,4,4,6,6,8,10,14,18,24,29,36,44,58,72,
91,113,143,179,227,287,366,460,578,732,926,1174,
1489,1879,2365,2988,3780,4788,6049,7628,9635,12190,
15409,19452,24561,31025,39229,49580,62680,79255,100144,
126542,159930,202085,255455,322869,408002,515542,651407,
823238,1040490,1315036,1661989,...

now seem sufficiently numerous to more than fill two lines in the next edition of Sloane's *Handbook of Integer Sequences*. Have you observed any interesting properties of the sequence?

Yours sincerely,

Richard K. Guy.

RKG:1

Richard K. Guy.

φc N.J.A. Sloane
J.C. Lagarias