A 4214
A 4215

Scan

David Betts
4 NJB

Correspondence

4 pages

2 seqs

J. f91 ~~2 sequences to enter~~ Done A4214 A4215

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5th November 1976

Dr. N.J.A. Sloane,
Mathematics Research Center,
Bell Telephone Laboratories Inc.,
Murray Hill,
New Jersey, 07974,
U.S.A.

Dear Dr. Sloane,

I have just purchased your fascinating book "A Handbook of Integer Sequences" and should be grateful if you would keep me on your list for supplements.

There is a series which arises when considering the allowed quantum mechanical energy levels of a particle in a cubical box, and which is also known in number theory. The question is, which integers are not expressible as the sum of three squared integers? Actually, in the quantum mechanical case, zeros are not allowed and the series is then:

name: not the sum of 3 squares. No ref.

New seq, please enter →

1, 2, 4, 5, 7, 8, 10, 13, 15, 16, 20, 23, 25, 28, 31, 32, 37, 39, 40, 47, 52, 55, 58, 60, 63, 64, 71, 79, 80, 85, 87, 92, 95, 100 .... (more terms below) (I)

These numbers were found by laborious trial and error on a computer. I am not aware of a literature on this subject. However there is a related series, in which zeros are allowed, and which is known in number theory. It is

New, please enter

7, 15, 23, 28, 31, 39, 47, 55, 60, 63, 71, 79, 87, 92, 95, 103, 111, 112, 119, 124, 127, 135, 143, 151, 156, 159, 167 .... (II)

name: the sum of 4 squares. No ref.

It has been proved (cf Dickson's "History of the Theory of Numbers") that all terms in the series are expressible in the form $4^a(8b+7)$ where a and b are non-negative integers. A further theorem (Wagstaff, Proc. Am. Math. Soc. 52, 1 (1975)) shows that

$$N(x) \leqslant \frac{x-1}{6} \quad \text{for} \quad x \geqslant 1$$

/Over ...

where N(x) is the number of terms in the series which are not greater than x.

I hope this may be of interest.

Yours sincerely,

David Betts

David S. Betts (Dr.)
Lecturer in Experimental
Physics

November 30, 1976

Dr. D. S. Betts
School of Physical Sciences
University of Sussex
Falmer
Brighton BN1 9QH
ENGLAND

Dear Dr. Betts:

Thank you very much for your letter of 5th November. That is the kind of letter an author likes to receive. A copy of Supplement I (the only one issued so far) is enclosed, as well as a couple of other things that may interest you.

Thank you for suggesting the two new sequences. By the way, you said that the first sequence, the members not expressible as the sum of three nonzero squares, was found by laborious trial and error on a computer. Won't the following method generate these numbers rather quickly? The nonzero squares are the exponents in

$$\phi(x) = x + x^4 + x^9 + x^{16} + x^{25} + x^{36} + \dots .$$

The numbers which are the sums of three nonzero squares are the exponents in

$$\phi(x)^3 = x^3 + 3x^6 + 3x^9 + 3x^{11} + x^{12} + 6x^{14} + 3x^{17} + \dots .$$

The numbers which don't appear as exponents are

1, 2, 4, 5, 7, 8, 10, 13, 15, 16,...

and there is your sequence. I wrote a trivial computer program (using MACSYMA) which generated the enclosed sequence.

If this problem is important one could probably find some theoretical properties of the sequence - is it?

With best regards.

Yours sincerely,

MH-1216-NJAS-mv

N. J. A. Sloane

Enc.
As above