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Dear Neil:

I recently came across a paper which contains a sequence of numbers which you do not have in your handbook of integer sequences (nor in the supplement), and I thought you might like to know about it.

It occurs in the paper by M.K. Gol'dberg and E.M. Livshits which appeared in Matematicheskie Zametki, Vol. 4 No. 3, Pgs. 371-379 (1968); translated in Mathematical Notes, Vol. 4 (1968), pgs. 173-177. This paper gives the first 15 terms of the sequence as 1,2,4,7,12,18,28,39,55,74,100,127,167,208,261. The calculation of these numbers is extremely simple and in case you should want further terms in the sequence I have calculated the first 99 of them (I meant to calculate a round 100, but overlooked the fact that the first term was a zero!). I give these numbers on an enclosed sheet of paper.

I have not forgotten about the sequences that I said I would send you - the ones concerning dissections of a polygon, but I am expecting some comments from Frank Harary and Ed Palmer, who are in England, and it is possible that, with these comments, will come some additional data. I will tell you one thing however, just to whet your appetite. The sixth term (the last one given) in sequence 339 is incorrect; it should be 75 not 73.

I see from Mathematical Reviews that you recently wrote a paper on finding the paths through the network (Bell System Tech. J. 51(1972), 371-390). I would greatly appreciate receiving a reprint of this paper if you have one to spare.

Best wishes.

Yours sincerely,

Ron Read

R.C. Read

RCR:wj

Thanked ✓

N394.5 = 3318

0	1	2	4	7	12	18	28	39	55
74	100	127	167	208	261	322	399	477	581
686	820	967	1142	1318	1545	1778	2053	2347	2697
3048	3486	3925	4441	4986	5610	6250	7024	7799	8680
9604	10673	11743	13008	14274	15718	17239	18937	20636	22623
24621	26872	29203	31812	34422	37411	40432	43814	47303	51144
54986	59401	63817	68673	73692	79227	84808	91088	97369	104277
111363	119150	126938	135687	144437	153963	163777	174475	185210	197109
209009	222039	235363	249758	264154	280139	296207	313542	331230	350373
369517	390637	411808	434680	457993	483006	508129	535678	563228	592818

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Gol'dberg M.K. & Livšic, E.M.

Minimal Universal Trees

MR 2049 p 384 Sept 1969 vol 38

Mat. Zametki 4 (1968) 371-379

(translation by L A Shepp and NJAS)

Let us consider connected oriented trees B with a root x_0. No directed edges run into x_0, while exactly one directed edge runs into every other node of B.

Let a_n be the class of all such trees that have at most n nodes.

Let $A, B \in a_n$. Say $B \subset A$ if A contains a subgraph B' isomorphic to B, whose root coincides ~~something~~ with the root of A

A tree U is called n-universal, if for every tree $A \in a_n$ we have $A \subset U$.

In this paper an n-universal tree with the min no $\alpha(n)$ of nodes is constructed. $\alpha(n)$ satisfies the recurrence

$$\alpha(1) = 1$$

$$\alpha(n) = 1 + \alpha(n-1) + \alpha\left(\left[\frac{n-1}{2}\right]\right) + \cdots + \alpha\left(\left[\frac{n-1}{n-1}\right]\right)$$

Also $\ln \alpha(n) \sim \frac{\ln^2 n}{(2 \ln 2)}$.

...

. . .

$\alpha(1) = 1$

$\alpha(2) = 1 + \alpha(1) = 2$

$\alpha(3) = 1 + \alpha(2) + \alpha(1) = 1+2+1 = 4$

$\alpha(4) = 1 + \alpha(3) + \alpha(\frac{3}{2}) + \alpha(1) = 1 + 4 + 1 + 1 = 7$

$\alpha(5) = 1 + \alpha(4) + \alpha(2) + \alpha(1) + \alpha(1) = 12$

. . .