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May 6, 1980

Ms. Susan Worst
14340 Woodbridge C.
Brookfield, Wisconsin 53005

Dear Ms. Worst:

Thanks very much for your letter of April 16 and the enclosed paper. Congratulations on winning that award! I read the paper and it looks like you have probably found the right answer (for the highest possible persistence). But proving it will be a much much harder problem. I just cannot see how to do it.

Good luck with your career! I enclose a few papers of mine that might interest you.

Very best regards,

MH-1216-NJAS-mv N. J. A. Sloane

Enc.
As above

April 16, 1980

Dear Mr. Sloane,

I just wanted to thank you for the help you gave me with my science fair project, Multiplicative Persistences of Base Four Numerals, when I called you last month.

My project took a first place award at our school's (Brookfield Central HS, Brookfield, Wisc.) science fair, making me eligible for the regional fair, held at Marquette University March 28-30. My project took first prize in the mathematics division there.

Going to Marquette was a marvelous experience and I'd just like to thank you for inventing multiplicative persistence in the first place, enabling me to become richer in both money ($50 bond from the Milwaukee Journal) and experiences.

Enclosed is the copy of my report that you requested. Thanks again!

God bless,

Susan Worst

(4340 Woodridge Ct.
Brookfield, Wisc. 53005)

Central Science Students Winners at Annual Fair

Five first place awards were made to science students in each of two categories at Brookfield Central High School's annual science fair held March 7 and 8 at Central.

Those taking first in the Biological Science Division were Andrew Lee, "The Effectiveness of ATP in Protecting Fresh Water Protozoa from Pesticides"; Thomas Aaberg, "Effects on Active Transport"; Jill Larson, "Inhibiting Effect of Fluorides on Bacterial Acid Production"; Debbie Saliba, "The Effect of Cytokinins on Leaf Senescence"; and Dan Peterson, "Effects of Contingent and Non-Contingent Verbal Reinforcement on Mathmatical Learning."

In the Physical Science Division, Joe Kronsnoble took a first for his project entitled "How the Wad Affects the Shot Pattern"; Tracy Trieglaff, "Woods' Heat of Combustion"; Lauri Scheffel, "Effects of Various Additives on Coagulants"; Kirk Wooldridge, "Are Strapless Grips Superior?"; and Susan Worst, "Persistance of Base-4 Numerals."

Seconds in the Biological Science Division went to Diane Carco, Mike Dolister, Bruce Urban, Leslie Kauffman, Angela Nohe and Sean Smullen; in the Physical Sciences Division to Todd Colin, Jeff Gibbs, Wallis Hoyle, Tim Goltz, Mark Nelson and Rob Ziobro.

In Biological Sciences, thirds went to Tom Merkel, George Stejic, Martha Schauer, Fran Ruzicka, Sherry Murphy, Gail Miiler and Julia Rymut; in the Physical Sciences, Gary Kirst, Andy John, Dale Prokupek, Sue Schott, Mitch Foster, Brann Andersen and Beth Stanko.

Of those who won at Central's fair and of those who participated but did not place, 24 were chosen to enter their projects at the State Science Fair to be held at Marquette March 28-30.

Those entering the Marquette Fair include James Hunter, Robert Ziobro, Derrick Krause, Kurt Bechthold, Sue Schott, Tim Goltz, Leslie Kauffman, Joe Kronsnoble, Lauri Scheffel, Dan Peterson, Debbie Saliba, Jill Larson, Tom Aaberg, Bruce Urban, Mike Dolister, Andrew Lee, Wallis Hoyle, Kirk Wooldridge, Tracy Trieglaff, Susan Worst, Mark Nelson, Sean Smullen, Jeff Gibbs and Angela Nohe.

Persistence

MULTIPLICATIVE PERSISTENCES

OF BASE FOUR NUMERAI

My project deals with the multiplicative persistence of numbers in base four. The multiplicative persistence of a number is the number of steps needed to reduce the number to a single digit by multiplying the digits together. For example, to find the persistence of the base ten number 57, multiply 5 and 7, the product of which is 35. Then multiply 3 and 5 and get 15, and multiply 1 and 5 and get 5. Five is a single digit number, so the sequence stops there, and the persistence is 3. Another example: for the number 62, multiply 6 and 2 and get 12, and then multiply 1 and 2 and the product is 2. The persistence of 62 is 2, because it took two multiplications to reduce 62 to a one digit number. My project applies this principle to only base four numerals.

The concept of multiplicative persistence* was developed by Mr. N. J. A. Sloane of Bell Laboratories in New Jersey. Mr. Sloane used an extremely precise computer to determine the persistences of numbers in base ten up to 10^{50}, and he believes that no number in base ten has a persistence greater than 11. He also worked with numbers in base three, and determined that the second term of the persistence sequence for base three numbers is either zero or a power of 2. He also believes that all powers of 2 in base three greater than 2^{15} contain a zero, and that therefore the maximum persistence of any base three number is three. Although this has not been proved, it has been found to be true for numbers up to 2^{500}. Mr. Sloane did not, however, do any work with base four numbers.

* The adjective "Multiplicative" is used to distinguish this process from additive persistence, which is the number of steps needed to reduce a number to a single digit one by adding its digits.

For my project, I tested combinations of 2's and 3's in two through ten digit combinations. (Obviously, any number that contains a zero would be reduced to zero when the digits are multiplied, and so the persistence of any such number is one. Also, in any number containing a 1, the 1 would not change the product of the digits. For this reason I excluded number combinations containing 1's or zeros).

After multiplying the digits of each number together, the product had to be converted from base ten to base four.

Since, in any base x, x=10 ~~in base ten~~, x^2=100 ~~in base ten~~, x^3=1000 ~~in base ten~~, and so forth, it was relatively easy for me to construct a table matching base four numerals to their base ten counterparts. A portion of this table is shown below.

Base ~~4~~ 10 Number		Base ~~10~~ 4 Value
1	=	1
2	=	2
3	=	3
4	=	10
16	=	100
64	=	1000

I was also able to obtain base four values for the base ten numbers 20, 30, etc. by multiplying the appropriate base four values by 2 or 3.

To change a number from base 10 to base 4, I used the following procedure.

subtract the number from table closest to 243

$$
\begin{array}{rcll}
243 & & & \\
-192 & = & 3000 & \leftarrow 3000 \text{ in base } 4 = 192 \text{ in base } 10 \\
\hline
51 & & & \\
-48 & = & 300 & \leftarrow 300 \text{ in base } 4 = 48 \text{ in base } 10 \\
\hline
3 & & & \\
-3 & = & \underline{3} & \quad 3 \text{ in base } 4 = 3 \text{ in base } 10 \\
\hline
0 & & 3303 & \leftarrow \text{Sum of base four values}
\end{array}
$$

Keep going until the difference is zero

Therefore, the decimal number 243 can be expressed as 3303 in base four.

I soon discovered that the lowest number in base four of persistence one was 10 $= 4_{10}$, the lowest number of persistence two is 22 $= 10_{10}$, and the lowest of persistence three is 333. However, I found no number of a persistence greater than three.

After I had tested numbers of up to 9 digits, I noticed a pattern developing in some combinations. For example, the decimal equivilent of 2:2 would equal A, and adding another 2 (2:2:2) would produce B. However, multiplying four 2's would produce A, but with a zero added on the end, and multiplying five 2's produced B with a zero on the end. This pattern continued, and, since I saw no reason why it would not continue indefinitely, I eliminated that combination. This pattern could also be seen in all the other number combinations, except the ones with only one 2 and those with none at all. A mathematical explanation for this pattern could be that any combination with more than one two would be divisible by four in base 10, and therefore by 10 in base four.

Since I had eliminated all combinations except those with one two and those with none at all, I limited further calculations to those two. Mr. Sloane, in dealing with base 3 numerals, had only gone as far as 2^{15} in his calculations so I felt that carrying my calculations as far as 3^{25} would be sufficient. The largest number to have a persistence of 3 was 3^{10}; all numbers larger had persistences of two.

Since the more digits a number has, the greater the chance that one of them is a zero, I feel that it is very likely that every power of three above 3^{10} contains at least one zero. Also, since the largest combination which contained a 2, and had a persistence of three was 2.3^{11}, it is very likely that every power of three x 2 above 2.3^{11} contains at least one zero.

If this is true, then the maximum persistence of any base four number is three. Unfortunately I can not do more than conjecture, since I have not found any way to prove it.

There is much more work that could be done with persistences. First, a computer with at least 20 significant figures could be used to verify my conjecture. Also, if indeed this is possible, a formal proof could be devised.

The concept of multiplicative persistences is a fascinating one, and it could be expanded to include many variations. For instance, the digits of a number could be multiplied and then the digits of the product added, or squared and then added. Division, however, would probably not work as a part of a persistence sequence, because you could not always be sure of getting a whole number for an answer.

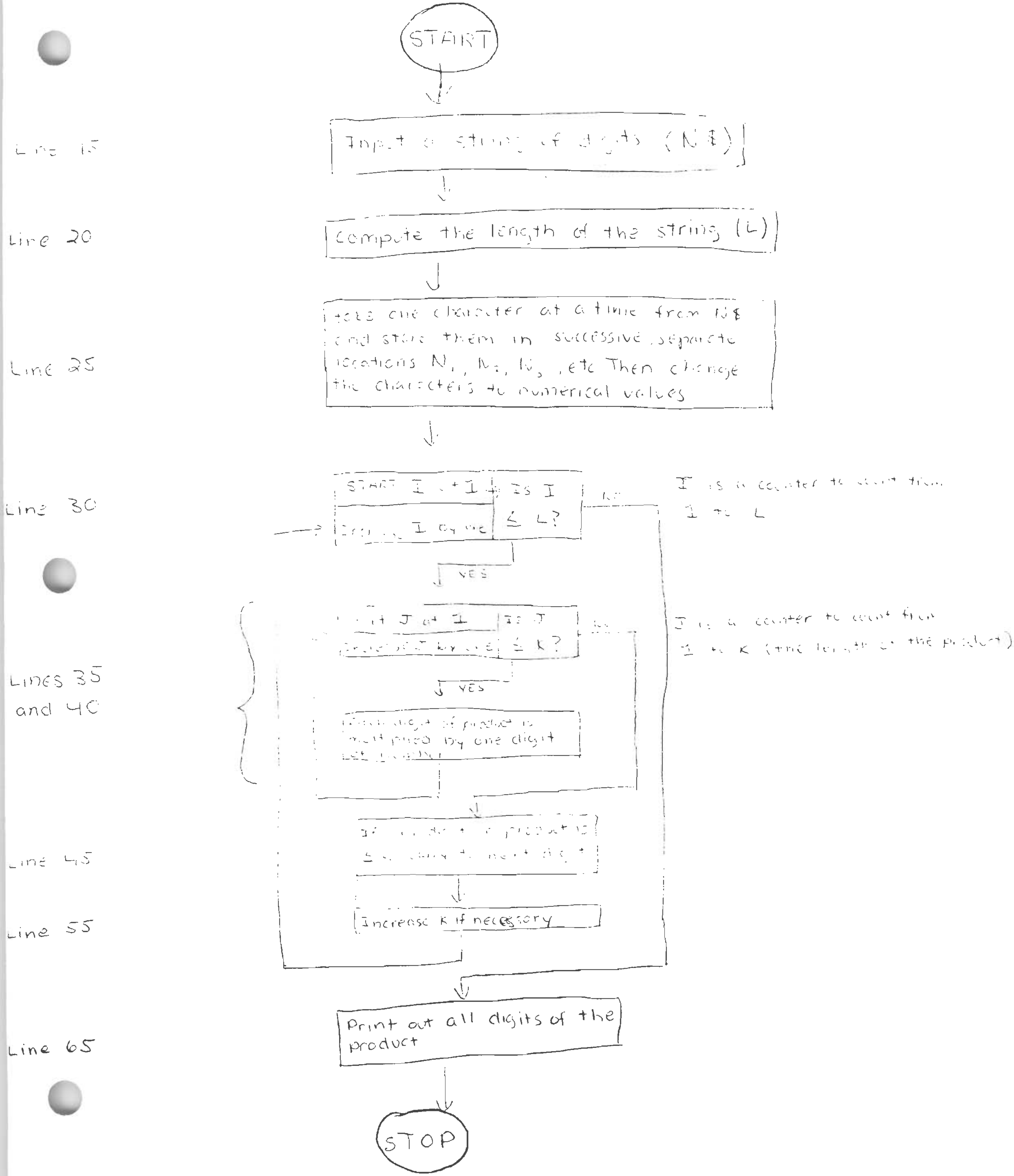
The value of this project is in its contribution to the knowledge of this particular aspect of number theory, rather than for practical purposes.

After the Brookfield Central High School Science Fair I was able, with the help of Central's Computer Science teacher, Mr. Hilmer, to write a computer program that could be used to multiply the digits of any base four number and convert the product to base four. The program is as follows:

```
2    Clear
5    DIM N (50), P (50)
10   P (1) = 1 : K =1
15   Print "Base Four Number?" : Input N $
20   L = LEN (N$)
25   For I =1 to L: N (1) = VAL (MID$(N$,I,1)):
     NEXT
30   For I = 1 TO L
35   For J =1 to K: P (J) = P (J) X N (I): NEXT
40   For J = 1 TO K
45   IF P (J) > 3 THEN P (J) = P (S)-4: P(J+1)=
     P (J=1) =1:  GO to 45
50   NEXT
55   If P (K=1) > 0 THEN K = K + 1
60   NEXT I
65   FOR I = K to 1 STEP - 1: PRINT P (I); : NEXT
70   PRINT ""PRINT"": GO TO 2
```

I used this program to text digit combinations of up to 3^{50}, but all of the resulting products contained a zero. This further verifies my hypothesis.

Program


START
Line 15
Line 20
Compute the length of the string (L)
Line 25
Line 30
Lines 35 and 40
Line 45
Line 55
Increase K if necessary
Line 65
Print out all digits of the product
STOP

Source

Scientific American, February, 1979

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Mr. N.J.A. Sloane, Bell Laboratories, for the same.

Mr. Hilmer, Computer Science Dept., Brookfield Central High School, for help with the computer program.