

2961

February 10, 1972

Professor David Singmaster
Poly of the South Bank
Borough Road
London (SE1), England

Dear Professor Singmaster:

John Brillhart suggested that I write to you about this. I am interested in the sequence of numbers n such that $\sigma(n) = \sigma(n+1)$, where $\sigma(n)$ is the sum of the divisions of n . Sierpinski (A Selection of Problems in the Theory of Numbers, p. 110) seems to say that the sequence begins 14, 206, 957, 1334, 1364, 1634, 2685, 2974, 4364. Could you possibly suggest a reference where I could find further terms (the next 20 or so), or could you even please supply these terms?

Yours sincerely,

MH-1216-NJAS-bk

N. J. A. Sloane

Copy to
Mr. W. O. Baker

APPROVED:

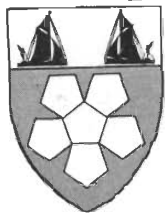
H. O. POLLAK



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Head: J M Dubbey MSc PhD FIMA

N. J. A. Sloane.

your ref

our ref

I have just happened to look in
Sierpinski; Elementary Theory of Numbers;
Translated by A. Hulanicki; Państwowe
Wydawnictwo Naukowe, Warszawa, 1964
(Probably there is a western publisher also?) on
p. 186. "The equation $\sigma(n) = \sigma(n+1)$
has only 9 solutions for $n < 10000$. These are
 $n = 14, 206, 957, 1334, 1364, 1634, 2685, 2974, 4364$
(cf. Makowski [3]). We do not know whether there
exist infinitely many natural numbers n for
which $\sigma(n) = \sigma(n+1)$.

A. Makowski has asked whether for
every integer k there exists a natural number n

such that $\sigma(n+1) - \sigma(n) = k$, and,
more generally, whether for every natural number
 m and every integer k there exists a natural
number n such that $\sigma(n+m) - \sigma(n) = k$. "

From the Bibliography
Malowski [3]: O pewnej funkcji liczbowej,
Matematyka 10 (1959) pp. 145-157

Presumably this is in Polish since his
references seem to give titles in the original
language, but I don't know the journal
unless it is ~~Matematyka~~ *Mathematika* (London)

Good luck and I hope this comes
in time to save you looking through
Glaisher.

Sincerely,

David Singmaster

Replies to:

Borough Road
London S.E.1

01-928 8989

Your ref:

Our ref:

31 May, 1972

N.J.A. Sloane

Sorry to have taken so long to respond,
I don't have the desired information at hand.
You can, with patience, get ~~some~~ (further?)
solutions from Glaisher's Number-Divisor Tables
(British Association for the Advancement of Science. Mathematical
Tables, Volume VIII. 1940, reprinted 1966 by
Cambridge University Press.) His Table I gives
 n and $\sigma(n)$ for $n=1(1)10000$. His Table IV
gives x and $\sigma^{-1}(x) = \{n \mid \sigma(n) = x\}$
for $x = 1$ ^{to} ~~(1)~~ 10000. Further, one can readily
compute $\sigma(n)$ for $1(1)100000$ or 10^6 .
I once computed a table of number theoretic
functions to 10^5 and it only took 20 min.
on a 7090 in 1963. I see no reason why

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one couldn't compute $\sigma(n)$ $n = 1(1)10^6$
and check against $\sigma(n-1)$ in about 10
minutes on a modern machine. I had indeed
thought of doing it, but have not had time.
If you still want the values, let me know
and I will be able to do them over the
summer.

I mentioned to Erdős ~~the~~ your interest
and he said he didn't know any examples
and he had once considered trying to showing
the existence of infinitely many n with
 $\sigma(n) = \sigma(n+1)$ but had considered it
too hard. I believe he considered $\phi(n) = \phi(n+1)$
and similarly rejected it.

Sincerely
David Singmaster