2961

February 10, 1972

Professor David Singmaster
Poly of the South Bank
Borough Road
London (SE1), England

Dear Professor Singmaster:

John Brillhart suggested that I write to you about this. I am interested in the sequence of numbers n such that $\sigma(n) = \sigma(n+1)$, where $\sigma(n)$ is the sum of the divisions of n. Sierpinski (A Selection of Problems in the Theory of Numbers, p. 110) seems to say that the sequence begins 14, 206, 957, 1334, 1364, 1634, 2685, 2974, 4364. Could you possibly suggest a reference where I could find further terms (the next 20 or so), or could you even please supply these terms?

Yours sincerely,

MH-1216-NJAS-bk

N. J. A. Sloane

THIS COPY FOR

Copy to
Mr. W. O. Baker

APPROVED:

H. O. POLLAK

New Seq/ 2961

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N. J. A. Sloane.

your ref our ref

I have just happened to look in Sierpinski; Elementary Theory of Numbers; Translated by A. Hulanicki; ~~W~~ Państwowe Wydawnictwo Naukowe, Warszawa, 1964 (Probably there is a Western publisher also?) on p. 166. "The equation $\sigma(n) = \sigma(n+1)$ has only 9 solutions for $n < 10000$. These are $n = 14, 206, 957, 1334, 1364, 1634, 2685, 2974, 4364$ (cf. Makowski [3]). We do not know whether there exist infinitely many natural numbers $n$ for which $\sigma(n) = \sigma(n+1)$.

A. Makowski has asked whether for every integer $k$ there exists a natural number $n$

such that $\sigma(n+1) - \sigma(n) = k$, and, more generally, whether for every natural number $m$ and every integer $k$ there exists a natural number $n$ such that $\sigma(n+m) - \sigma(n) = k$."

From the Bibliography

Makowski [3]: O pewnej funkcji liczbowej, Matematyka 10 (1959) pp. 145-157

Presumably this is in Polish since his references seem to give titles in the original language, but I don't know the journal unless it is ~~Matematika~~ Mathematika (London)

Good luck and I hope this comes in time to save you looking through Glaisher.

Sincerely,

David Singmaster

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POLYTECHNIC OF THE SOUTH BANK

Department of Mathematics

Head:
Jon V. Pepper
M.A., M.Sc., F.I.M.A.

Replies to:
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31 May, 1972

N.J.A. Sloane

Sorry to have taken so long to respond. I don't have the desired information at hand. You can, with patience, get ~~some~~ (further?) solutions from Glaisher's Number-Divisor Tables (British Association for the Advancement of Science. Mathematical Tables, Volume VIII. 1940, reprinted 1966 by Cambridge University Press.) His Table I gives $n$ and $\sigma(n)$ for $n = 1\,(1)\,10000$. His table IV gives $x$ and $\sigma^{-1}(x) = \{n \mid \sigma(n) = x\}$ for $x = 1$ to $10000$. Further, one can readily compute $\sigma(n)$ for $1\,(1)\,100000$ or $10^6$. I once computed a table of number theoretic functions to $10^5$ and it only took 20 min. on a 7090 in 1963. I see no reason why one couldn't compute $\sigma(n)$ $n = 1(1)10^6$ and check against $\sigma(n-1)$ in about 10 minutes on a modern machine. I had indeed thought of doing it, but have not had time. If you still want the values, let me know and I will be able to do them over the summer.

I mentioned to Erdös ~~the~~ your interest and he said he didn't know any examples and he had once considered trying to showing the existence of infinitely many $n$ with $\sigma(n) = \sigma(n+1)$ but had considered it too hard. I believe he considered $\phi(n) = \phi(n+1)$ and similarly rejected it.

Sincerely

David Singmaster