is: the output value of the

\[ -1)w_e > 0 \]

(85)

\[ -1)w_e < 0, \]

the input vector \((j = 1, \ldots, 2^n)\)

of a majority decision

to the linear programming

\[ 1 + \cdots + w_e \]

is to be inequalities (85) where equalities knowing the minimum of

the maximum of those of the

inequalities in (85) do not

small \(w_e\)'s and \(w_e\) can be solu-

minimization problem. Then

first inequalities and the maxi-

mizes could be known or pre-

performance of the element. \(w_e\) and write it as \(e\), that is, \(w_e\) can be made to include

by dividing these by \(e\).

of Problem 1 and the simplex

the same processes as those

easily seen. This form of the

working directly on \(w_e\) without

to see that if we introduce a

decision element given by

the constraints in Problem 3

he objective function also will

des formally with Problem 1

non-integral coupling numbers

to solve Problem 1 formally.

blem 1 the relation between \(T\)

to the threshold \(T\) only;

\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
\(n\) & \(2^n\) & \(n\) & \(2^n\) & \(n\) & \(2^n\) & \(n\) & \(2^n\) \\
\hline
1 & 4 & 1 & 1 & 1 & 1 & 1 & 1 \\
2 & 16 & 2 & 2 & 2 & 2 & 2 & 2 \\
3 & 65536 & 5 & 5 & 5 & 5 & 5 & 5 \\
4 & 4294967296 & 9 & 9 & 9 & 9 & 9 & 9 \\
5 & 783643 & 18 & 18 & 18 & 18 & 18 & 18 \\
6 & 444740737055616 & 994 & 994 & 994 & 994 & 994 & 994 \\
\hline
\end{tabular}

*Functions identical by permutations and negations of variables are counted into a single representative function.*
Majority Decision Functions of a Few Variables and Their Number

All majority decision functions of up to six variables are classified by permutations of variables, and negations of the variables. A representative function in each class such that \( w_1 \geq w_2 \geq w_3 \geq w_4 \geq w_5 \geq w_6 \geq 0 \) are obtained (16, 17). The number of these functions is too great to be shown here.

These functions were checked both by the simplex method and the combination method, where the functions are discovered by giving all possible integer values to the inputs.

Table II lists the numbers of functions enumerated from several different viewpoints (18).

(i) The number of general Boolean functions of up to \( n \) variables, \( 2^n \), is shown for reference in the second column of Table II.

(ii) The number of the positive functions of up to \( n \) variables, those that can be realized without negation of the variables. These may generally not be realizable by a single majority decision element. The numbers were quoted from Birkhoff’s book (19). The majority decision functions constitute only a subset of these.

(iii) The number of the majority decision functions of exactly \( n \) variables without negations of the variables. This is divided into two cases. The first case shows the number of the majority decision functions, representatives of equivalent classes, each of which consists of functions identical by permutations and negations of the variables. For example, \( x_1x_2 + x_1, x_1x_2 + x_2 \) and \( x_1 + x_2 + x_3 \) belong to the same class represented by \( x_1 + x_3x_2 \). The second case shows the number of the functions in the first case, taking into account only permutations of variables. The functions in the second case are not enumerable unless the forms of the functions representing the above classes are known, because there are partially or totally symmetric functions. For example, for \( n = 2 \), we have two, counting \( x_1 + x_2 \) and \( x_1x_2 \).

(iv) The number of the majority decision functions of up to \( n \) variables, without negations of variables. For example, for \( n = 2 \) we have six, counting 0, 1, \( x_1, x_2, x_1 + x_2 \) and \( x_1x_2 \).

(v) The number \( N(n) \) of the majority decision functions of up to \( n \) variables, taking into account permutations and negations of variables. If this is compared with \( 2^n \), we can see how small a part of \( 2^n \) this occupies.

(vi) Theoretical upper and lower bounds on \( N(n) \) shown in Section 4. The lower bound shows the number of functions constructed similarly to

May, 1961.}