Scan
4 pages

J. A. Sharp
and NYA —
Correspondence
A6347, 8

N.J.A. Sloane
Maths Research Centre
Bell Telephone Labs
Murray Hill
NJ 07974

fa

20 THE GLEBE 2026
GARSTON ~~[illegible]~~
WATFORD 6348
HERTS 6347
WD2 6LR
ENGLAND.

3/3/77

Dear Professor Sloane,

I should like to thank you for the many hours of enjoyment your Handbook of Integer Sequences has given me. I am a chemist by profession but enjoy spending a great deal of my time with recreational maths especially playing with numbers.

I should like to take you up on your offer of supplements of corrections etc. I should like to make a few contributions.

1) On page 18 the Catalan numbers : and form seq 557 should read 577

page 20 Seq 491 : 2, 4, 8, 15 should read 499.

2) Martin Gardner in his Mathematical Games column in the Scientific American introduced you to me, through the Catalan Numbers article last year. I played about with

them from a number difference point of view.

:-

1 2 5 14 42 132 429 1430 →
1 3 9 28 90 297
2 6 19 62 207 704
4 13 43 145 497
9 30 102
21
51

The series 1, 2, 4, 9, 21, 51, 127 etc is seq <u>456</u> generalised Ballot numbers

1, 3, 9, 28, 90, 297 are Laplace Transform coeffs <u>seq</u> 1130.

If the <u>456</u> sequence is again written as a difference table

1 1 2 4 9 21 51 127
→ 0 1 2 5 30 76 196 512 ← seq 554
1 1 3 7 18
0 2 4 11 28
2 2 7 17
0 5 10
5 5

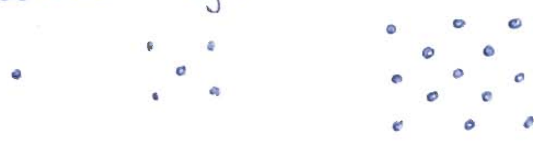
↑ ↖ "double" catalan numbers

Catalan nos separated by zeros.

1, 1, 2, 2, 5, 5, 14, 14, 42, 42 etc

2026 sequence 554 stops at 196 in the Handbook. 512, 1353 are apparently the next terms and more can be obtained from this difference

(3) Martin Gardner also mentions the star number sequence 1, 13, 37, 73, 121 in his column of July 1974. Meanwhile following other results I investigated central polygonal numbers eg: central square polygonal numbers form n^{th} by placing square around $(n-1)^{th}$ with n on each side



1844

1	5	13
n = 1	n = 2	n = 3.

The general term being $\left[1 + \frac{(r-1)rn}{2}\right]$

for rth term of n sided polygonal number.

for n = 1 you list them as seq <u>391</u> as central polygonal numbers

for n = 2 they are 1, 3, 7, 13, ie seq <u>1049</u> listed as central polygonal numbers again

n = 3 does not occur :- 1, 4, 10, 19, 31, 46, 64

n = 4 is sequence 1567 which is better written $n^2 + (n-1)^2$

n = 5 ; n = 6 are missing

n = 7 occurs as seq <u>1828</u> Cuban Primes

the only other one known in another context is n = 12 which gives 1, 13, 37, 73, 121 which are the star numbers mentioned above

(4) Finally I looked at analogous subfactorial numbers. Instead of
$$D_n = n D_{n-1} + (-1)^n \qquad \text{(base p 27)}$$

$1, 2, 9, 44,$ etc

for which $\displaystyle\lim_{n \to \infty} \frac{\lfloor n}{D_n} \to e$

6347 new

For $D_n = (n+1) D_{n-1} + (-1)^n$

gives $0, 1, 3, 16, 95, 666$ ----

giving $\displaystyle\lim_{n \to \infty} \frac{\lfloor n+1}{D_n} \to 7.568846$ ----

6348 new

For $D_n = (n+2) D_{n-1} + (-1)^n$

ie $1, 4, 25, 174, 1393$ ---

giving $\displaystyle\lim_{n \to \infty} \frac{\lfloor n+2}{D_n} \to 28.9468$ ----

Are these related to e or a function of e? If so how? It struck me that a companion book to locate such numbers or other numbers such as functions of square roots π, e etc would be as useful as your Handbook.

With thanks once more

John A Sharp.

6347
6348

Bell Laboratories

600 Mountain Avenue
Murray Hill, New Jersey 07974
Phone (201) 582-3000

March 22, 1977

Dr. J. A. Sharp
20 The Glebe
Garston
Watford
Herts
WD2 6LR
ENGLAND

Dear Dr. Sharp:

Thank you very much for your letter of March 3 and the kind words about the Sequence Handbook. I am glad you liked it. Thank you also for the new sequences and suggestions for extending old sequences. Very helpful.

Your generalized subfactorial sequences are also interesting. First, consider the original subfactorial sequence $\{D_n\}$ with $D_1 = 0$, $D_2 = 1$, $D_3 = 2$, and

$$D_n = nD_{n-1} + (-1)^n, \quad n \geq 2.$$

One quickly sees after writing out a few terms that

$$D_n = n!(1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \ldots + (-1)^n \frac{1}{n!})$$

The expression in brackets is very close to the Taylor series for e^{-1} if n is large, and hence

$$\lim_{n\to\infty} D_n/n! = \frac{1}{e}.$$

Similarly if $D_1 = 0$ and

$$D_n = (n+1)D_{n-1} + (-1)^n$$

then

$$D_n = (n+1)!\ (\frac{1}{3!} - \frac{1}{4!} + \ldots + (-1)^n \cdot \frac{1}{(n+1)!}).$$

Therefore

$$\lim_{n\to\infty} D_n/(n+1)! = -\frac{1}{e} + \frac{1}{2}.$$

And in general if $D_1 = 0$ and

$$D_n = (n+r)\ D_{n-1} + (-1)^n$$

then

$$D_n = (-1)^r\ (n+r)!\ \left\{e^{-1} - (\frac{1}{2!} - \frac{1}{3!} + \ldots + (-1)^{r+1} \cdot \frac{1}{(r+1)!})\right\}$$

so that

$$\lim_{n\to\infty} D_n/(n+r)! = (-1)^r \left\{\frac{1}{e} - \frac{1}{2!} + \frac{1}{3!} - \ldots + (-1)^r \cdot \frac{1}{(r+1)!}\right\}.$$

Do you agree?

Yours sincerely,

MH-1216-NJAS-mv

N. J. A. Sloane