Dear Neil,

Please send me a copy of your "Handbook of Integer Sequences" I saw the review in S.A. April 1974.

Yes, I was heartily surprised at how many sequences have convolution arrays with the limit determinant property. All (1, 1, x, y) sequences do x, y, z, ... are arbitrary. The proof of the theorem (Hoggatt-Kramer) is in a thesis of Judy Kramer available from the Fibonacci Association (Brother Alfred above) for $10.00.

Enclosed is the paper by Hoggatt & Bergum.

Most sincerely,

Sincerely,
Dear Neil,

April 20, 1974

I see from your format you can insert new sequences endlessly with references for each one. I appreciate your Supplement I. I'll be happy to send you several (hundred) more sequences, but alas I'll have to wait until I see your general plan and format. (ad what you already have. I eagerly await your handbook via Academia Press.

In two of our publications edited by Brother Alfred there are several sequences too! When I get a few days I'll do you some real good!!

For instance: 1, 1, 2, 5, 14, 42, ... CATALAN NUMBERS

\[
\frac{1}{n+1} \binom{2n}{n}
\]

However 1, 2, 5, 14, 42, ... is the FIRST CONVOLUTION SEQUENCE FOR THE CATALAN NUMBERS.

Reference (Appendix) Sequence 1174 1, 3, 12, 55, ...

This the second Convolution sequence for 1, 1, 3, 12, 55...

Your sequence 1174 appears in Carlet's article "Two-line Enumeration Your sequence 1174 appears in Carlet's article "Two-line Enumeration"

I'll have much more to say on all this.

Sincerely, Vern
Dear Neil,

You almost threw me when you asked about

\[1, 1, 5, 12, 28, 64, \ldots \quad A_{n+1} = 2A_n + 2^{n-2}\]

This occurs in the country the compositions of \(N\) using all positive integers as possible summands. If one then displays all such compositions all the same time the number of ones used is given by the sequence above.

\[
\begin{align*}
1 &= 1 \\
2 &= 2, 1+1 \\
3 &= 2+1, 1+2, 1+1+1, 3 \\
4 &= 2+1+1, 3+1, 1+1+1+1 \\
&\quad 1+2+1, 1+3, 4 \\
&\quad 1+1+2, 2+2 \\
5 &= 2+2+1, 2+1+1+1, 1+2+1+1, \ldots \\
&\quad 3+1+1+1, 1+3+1, 4+1, 1+1+1+1+1 \\
&\quad 1+2+1+1, 1+1+2+1, 1+1+1+2 \\
&\quad 3+2, 2+3
\end{align*}
\]

This is also the convolution of 1, 1, 2, 4, 8, \ldots with itself. The reference is "Palindromic Compositions" by V.E. Hoggatt, Jr. and Marjorie Bicknell to appear in...
The Fibonacci Quarterly Dec. 1975, I cannot tell you the page numbers in that issue yet.

Yes, Neil, I am well aware of my promise. Since by far most of my articles are written with co-authors Marjorie Bicknell, I'll try to get her to "cooperate" and get you some new sequences real soon. I get new sequences almost every day.

N.B. If \( F(x) = x^{a_0} + x^{a_1} + \ldots + x^{a_k} + \ldots \)

then the number of compositions of \( N \) using summands from \( \{a_n\}_{n=0}^\infty \) is given by

\[
\sum_{n=0}^{\infty} C_n x^n = \frac{1}{1 - F(x)}
\]

In case \( F(x) = x + x^2 + x^3 + \ldots + x^n + \ldots = \frac{1}{1-x} \) then the generating function is \( \frac{1-x}{1-2x} \).

Now the number of occurrences of 1 \( \in \{1, 1, 2, 1, 3, 2, 4, \ldots \} \) in all the composition of \( N \) is given by

\[
\frac{x}{(1 - F(x))^2} = \frac{x(1-x)^2}{(1-2x)^2}
\]
the convolution of sequence 1, 1, 2, 3, 5, ... with itself.

Sincerely, Vern

P.S. The occurrence of \( k \) (a positive integer in the composition of \( n \)) is given by

\[ x^k \left( \frac{(1-x)^2}{1-2x} \right) \]

which is the same sequence with \( (k-1) \) zeros at the front. Thus the sequence is well used in this particular problem.
Dear Neil (N.J.A. Sloane) Nov. 14, 1976

If you have watched the Fibonacci Quarterly you will have seen: FIVE ARTICLES

1) The H-Convolution Transform.

APRIL 1976

2) PASCAL, CATALAN and General Sequence Convolution

March 1976

3) Anamorphic Matrix Arrangements

October 1976

4) Sequence Inverses from Pascal CATALAN and Related Convolution Arrays.

Dec 1976

5) CATALAN and Related Sequences arising from inverses of Pascal's Triangle Matrices.

Feb 1977

6) Numerator Polynomial Coefficient Arrays for CATALAN and Related Sequence Convolution Triangles.

February 1976/7. Numerator Polynomial Coefficient Array for convolved Fibonacci Sequence.

These articles are chock-full of sequences at some of which are not in your supplemental list. Actually I could send you a New Sequence everyday as I am in a creative resurgence right now. I would like to see a section on polynomial recurrence sequence. It would be messier but a great help in some researches. Every triangle...)
Actually carries an infinitude of sequences. Right now I'm picking Generalized Catalan Numbers from new Pascal-like Triangles.

**Catalan Numbers**

\[
C_1(x) = \frac{1 - \sqrt{1 - 4x}}{2x}
\]

\[
\frac{1}{\sqrt{1 - 4x}} = \{1, 2, 6, 20, 70, \ldots \}^2
\]

\[
C_2(x) = \frac{1}{1-x} C_1 \left( \frac{x}{(1-x)^2} \right) = \frac{1-x - \sqrt{1-2x + 3x^2}}{2x^2}
\]

\[
1, 3, 7, 19, 51, \ldots \ #1070 \ \text{central term} \ \text{Trinomial Triangle}
\]

\[
C_3(x) = \frac{1}{1-x} C_1 \left( \frac{x}{(1-x)^2} \right) = \frac{1-x - \sqrt{1-6x + x^2}}{2x}
\]

from the triangle above

\[
1, 2, 6, 22, 90, \ldots
\]

\[
= 14 \times 2 \ (1, 3, 11, 45, \ldots)
\]

This is in your table as #1163

Your #1174 occurs as a generalized Catalan lattice count. See fig 2 pg 136 April 1976 FQJ.

Also #1163 appears in Oct 1976 Page 231

\[
\sum_{n=1}^{\infty} b_n x^n = \frac{1 + x - \sqrt{1 - 6x + x^2}}{4x}
\]

Some dialogue, please!

Sincerely, Venā
Recurrence for column generators of table one is:

\[ f_{n+1} x^2 C_n^2 (x) = (1 - 2x) C_n^{2n+2} (x) - C_n^{2n+1} (x) \]

h2o

from which the entire table may be created once the coefficients of C(x) are known.

There

\[ C_{n+1} = \sum_{k=0}^{n} C_k C_{n-k} \]

C_0 = 1

will do it.
Dear Neil,

Thank you for the copy of Athanassios Papoulis’ “A new method of inversion of the Laplace Transform.”

I now affirm that those entries in Table One are indeed Convolution sequences for the Catalan Sequence:

\[
\begin{align*}
0 & \quad 1 \\
1 & \quad 1 \\
1 & \quad 1 \quad 2 \\
1 & \quad 3 \quad 3 \quad 1 \\
1 & \quad 3 \quad 4 \quad 2 \quad 6 \\
1 & \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \\
1 & \quad 5 \quad 6 \quad 9 \quad 15 \quad 5 \quad 20 \\
\end{align*}
\]

The circled numbers are by columns the Catalan Sequences \(C_n(x)\), the next left \(C^2_n(x)\), etc.
This appears, at least, in a thesis

The generating function for the vertical columns of the \((N+1)\)-gonal triangle by Michel Rondroux, written under my guidance in Dec 1973 at San Jose State University. Therein are many relations such as the

above.

Sincerely, Ven