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$$f(x) = \frac{1}{x \ln 2}, \qquad g(y) = \frac{1}{y \ln 2}$$

$$p = 1/2$$

here is nothing to choose between the two locations for the ry point[3] since the sum of the two probabilities is 1.

r division, the probability of a shift is also 1/2, regardless of the lution, since it is equally likely that the numerator will be r than or smaller than the denominator (though of course the al wiring may be slightly different in the two cases).

ne place where the choice does make a difference is in fixed- t double-precision multiplication by 1.000 · · · or 0.100 · · · . n the binary point is in front, then there is a loss of the last ficant digit on the right, whereas when the point is behind re is no such loss for any product. This is because the double- th accumulator usually has a guard position on the left end to care of overflow in addition.

n conclusion, there appear to be only slight engineering differ- s between the two choices as far as hardware is concerned. e humans learn binary arithmetic by analogy with the already rned decimal arithmetic, it seems that placing the binary point er the first digit should be considered for humane reasons al- ugh the change from current practice will cause a good deal of porary confusion.

[3] This is true for any positive integer base.

ngest "Separated" Paths and Loops in an N Cube

D. W. DAVIES

W. L. Black[1] describes two kinds of combination lock employing set of switches as input and a single-bit store. The operation of e form of lock can be described as follows. The state of the n itches corresponds to a point of an n cube. If one switch is moved t a time, the sequence of moves corresponds to a path on the n cube, ving from vertex to adjacent vertex along a cube edge. The single- it store is *set* at one end of the path and *cleared* if the state of the itches strays from the chosen path. The other end of the path is he goal. If it is reached with the single-bit store *set*, the lock opens.

An examination of W. L. Black's other kind of combination lock, nd a realization that to ensure that only one switch moves at a , the switches must be regarded as three state devices or addi- l storage provided, shows that the form of lock described above y not be the best practical one. But the study of paths on an cube suitable for such a lock is interesting in itself.

To avoid the possibility of cheating the lock, the states of the itches must correspond to a path on the n cube which never pproaches within a distance less than $\sqrt{2}$ of itself. Otherwise, the operation of a single switch could jump across the gap. More recisely stated, if $d(i, j)$ is the distance between the ith and jth oints on the path, which are numbered in sequence from $i = o$ to , where P is the number of edges in the path, then

$$d(i, j) = 1 \quad \text{for} \quad |i - j| = 1$$

$$d(i, j) > 1 \quad \text{for} \quad |i - j| > 1$$

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e author is with the National Physical Lab., Teddington, Middlesex, England.
Black, W. L., Electronic combination lock, *Quarterly Progress Rept. of the Research Lab. of Electronics*, No. 73, Massachusetts Institute of Technology, Cambridge, Mass., Apr 1964, p 232.

The interesting problem suggested by this lock is: what is the maximum length P of a path having this "separation" property for each value of n? In a similar way, a loop of length L can be defined, and we are interested in the longest "separated" loop.

A program was made for a Ferranti Hermes computer which obtained the answers by enumeration for values of n up to six. The number of different paths and loops of the maximum size was counted. The multiplicity of occurrences of paths and loops of maximum size due to permutation of axes was avoided. All the paths examined had the first occurrence of edges aligned with any of the n axes in a prescribed order. That is, the kth axis could not be used unless the axes $k-1$, $k-2$, $\cdots$, 1 had already been used. Any remaining unnecessary multiplicity would be due to describing the paths or loops in opposite directions or, in the case of loops only, starting the description at a different point. The results of the Hermes runs are given in the following Table I. For $n = 1$, the loop is 0–1–0.

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n	P_{max}	Number of paths	L_{max}	Number of loops
1	1	1	2	1
2	2	1	4	1
3	4	1	6	1
4	7	1	8	7
5	13	8	14	35
6	26	1	26	130

The path of length 26 on the 6 cube is remarkable. It is

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where the numbers specify the axes of the 6 cube with which successive edges of the path are aligned. The computer enumeration showed no other essentially different separated path of this length on the 6 cube.

This path was the only path or loop among those examined in which there was a spare vertex of the cube distant more than unity from all points of the path or loop. However, only one or two of the paths or loops of each set were examined, and others that were not examined might possibly have such an isolated vertex.

The results give rise to the conjecture that for n greater than 2, $L_{max}(n+1) = 2P_{max}(n)$. It can be further conjectured that longest separated loops can be formed from two separated paths of one lower dimension joined at the ends.

E. N. Gilbert[2] treated paths and loops on the n cube without the separation property. He regarded them as representing Gray codes for encoding angles and displacements with protection against gross errors due to ambiguity in the changing digits. The separated paths and loops correspond to particular Gray codes with redundancy which provides single error correction except, of course, when the error lies in one of the two digits which can validly change. A single digital error will at worst cause a small analog error. Paths and loops with a greater separation property can be defined and will give security against multiple errors.

These results are presented in the hope that they may stimulate a systematic theory of paths and loops with the separation property. They might find application in telemetry and in combination locks designed to guard against accidental operation in remote control.

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[2] Gilbert, E. N., Gray codes and paths on the n-cube, *Bell Syst. Tech. J.*, vol 37, May 1958, pp 815–826.