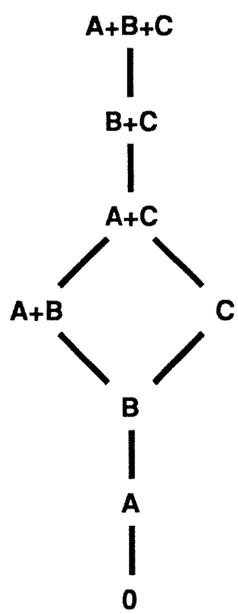
6 Partially Ordered Sets of Positive Sums

Given the relation 0 < A < B < C < D, there are obvious consequences. Transitivity insists that A < C, and B < D. And clearly B < A+B. But is A+D greater than or less than B+C? Without further information, these two sums are incomparable. Nor is A+B comparable to C.

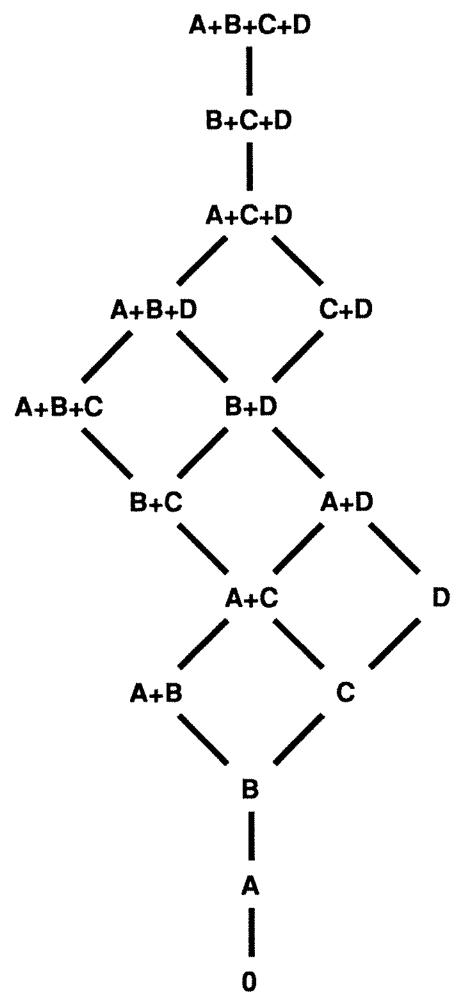
The figures below illustrate posets of positive sums with up to 7 elements. It is interesting to note the n-cube structures that appear in the centers of the figures, where the greatest ambiguity occurs. Note also that, if the single quantities — A, B, C, etc — are assigned the consecutive integers — 1, 2, 3, etc — then all sums on the same horizontal level are equal.

0 < A < B

0 < A < B < C

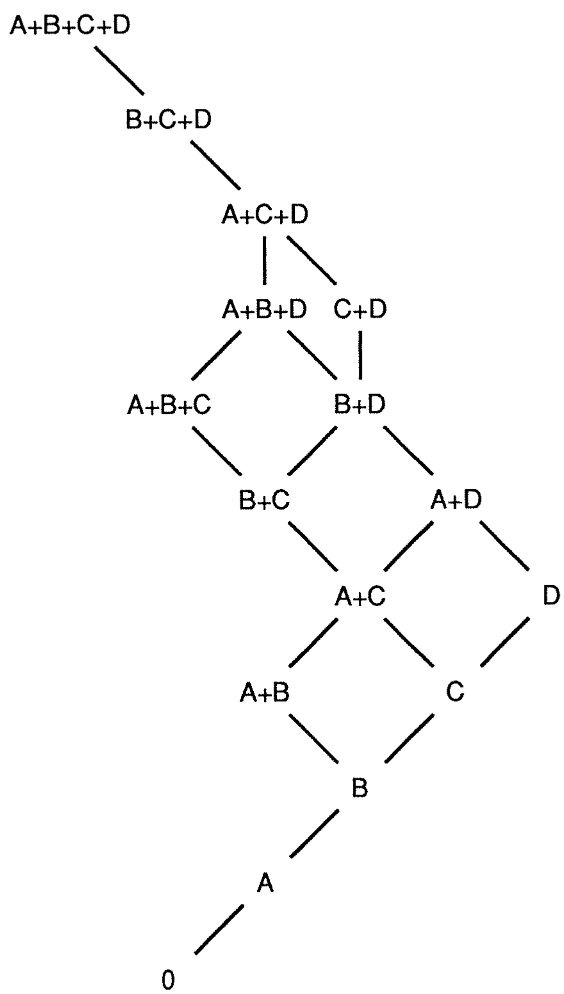


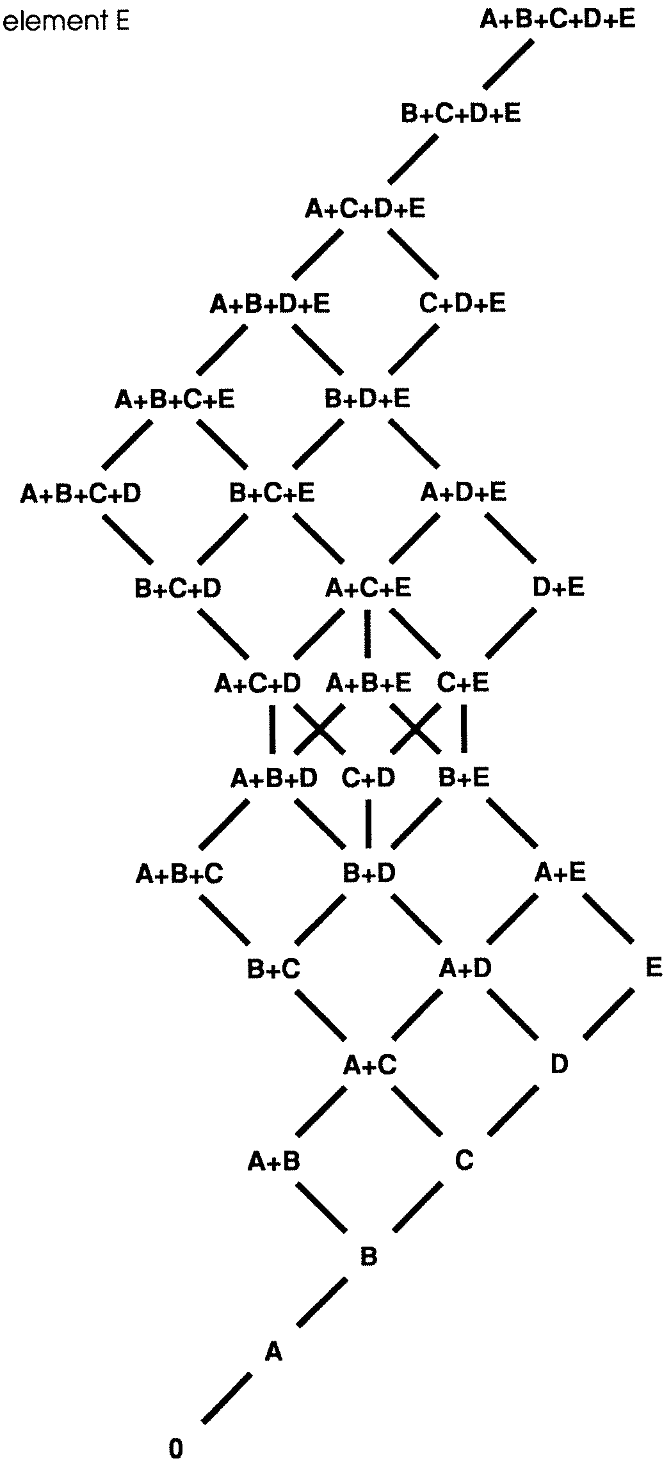
0 < A < B < C < D



0 < A < B < C < D < E

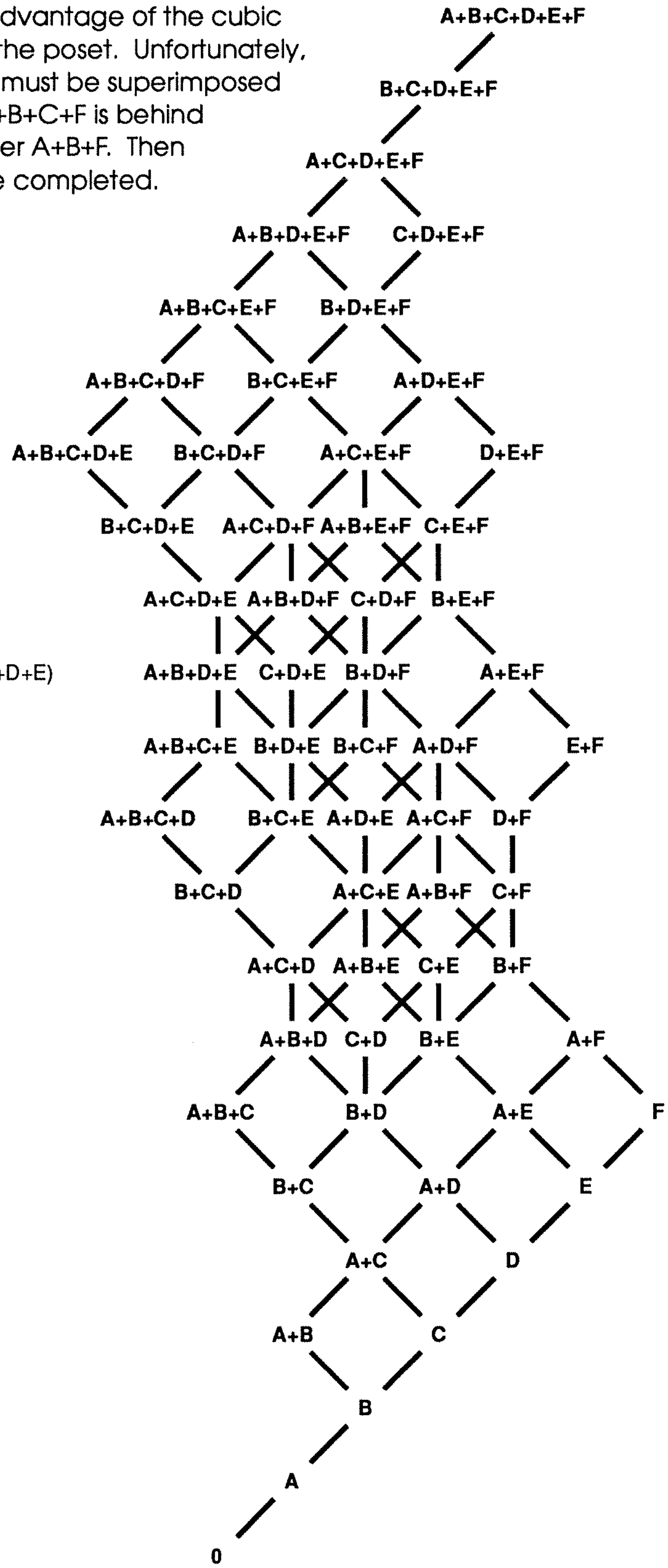
The light rendering on the left illustrates how the previous poset is embedded in the new one. The remainder of the poset is congruent to this one, where the new element E has been added to every entry.

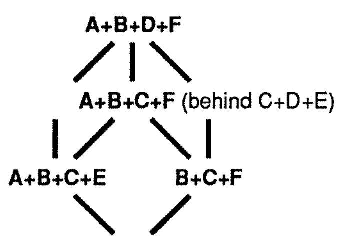


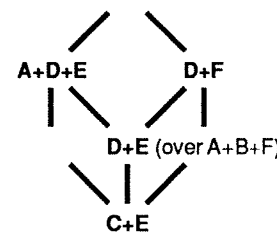


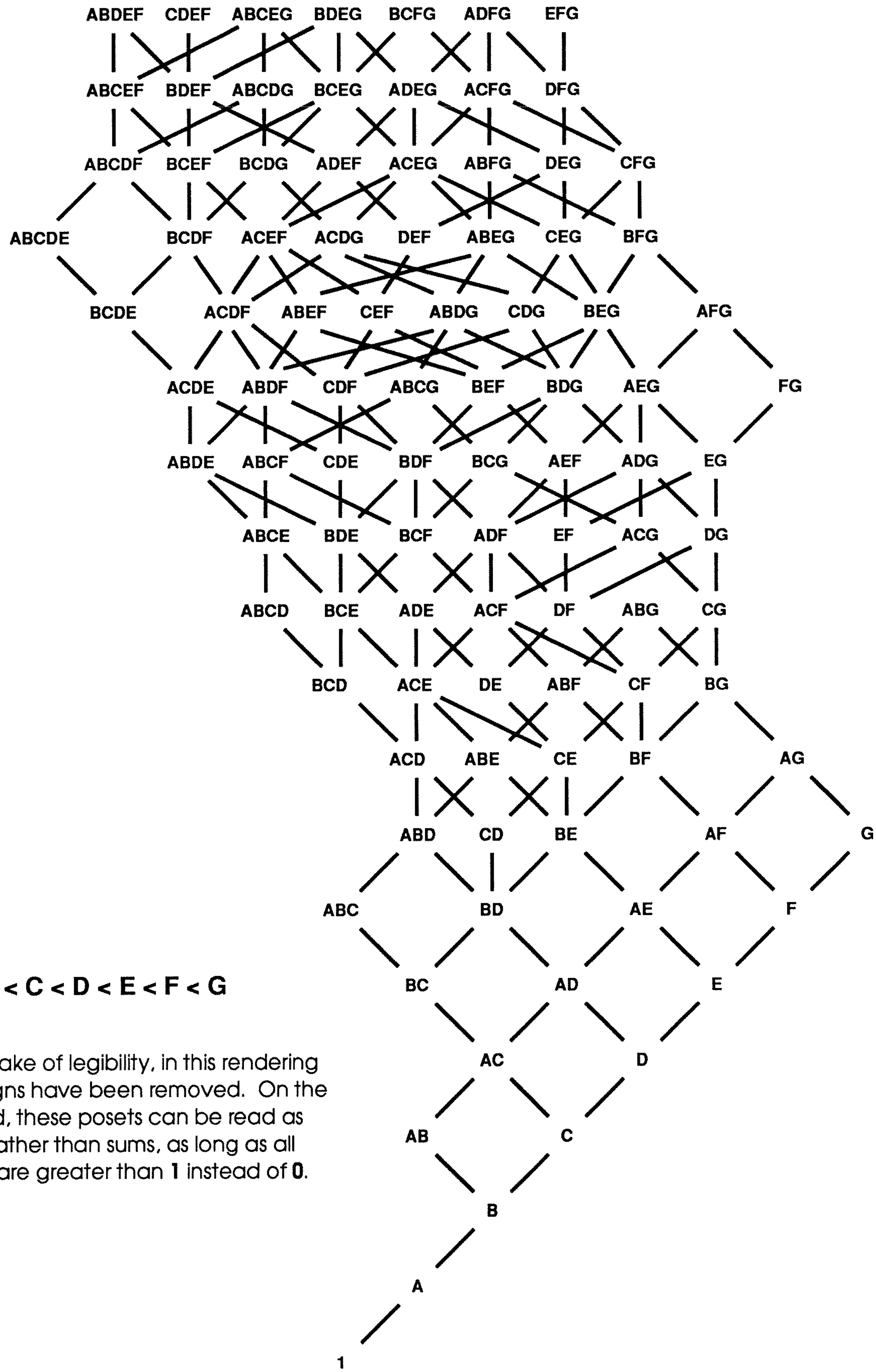
0 < A < B < C < D < E < F

This rendering takes advantage of the cubic lattice suggested by the poset. Unfortunately, in two places, entries must be superimposed on existing entries. A+B+C+F is behind C+D+E, and D+E is over A+B+F. Then 7 apparent cubes are completed.



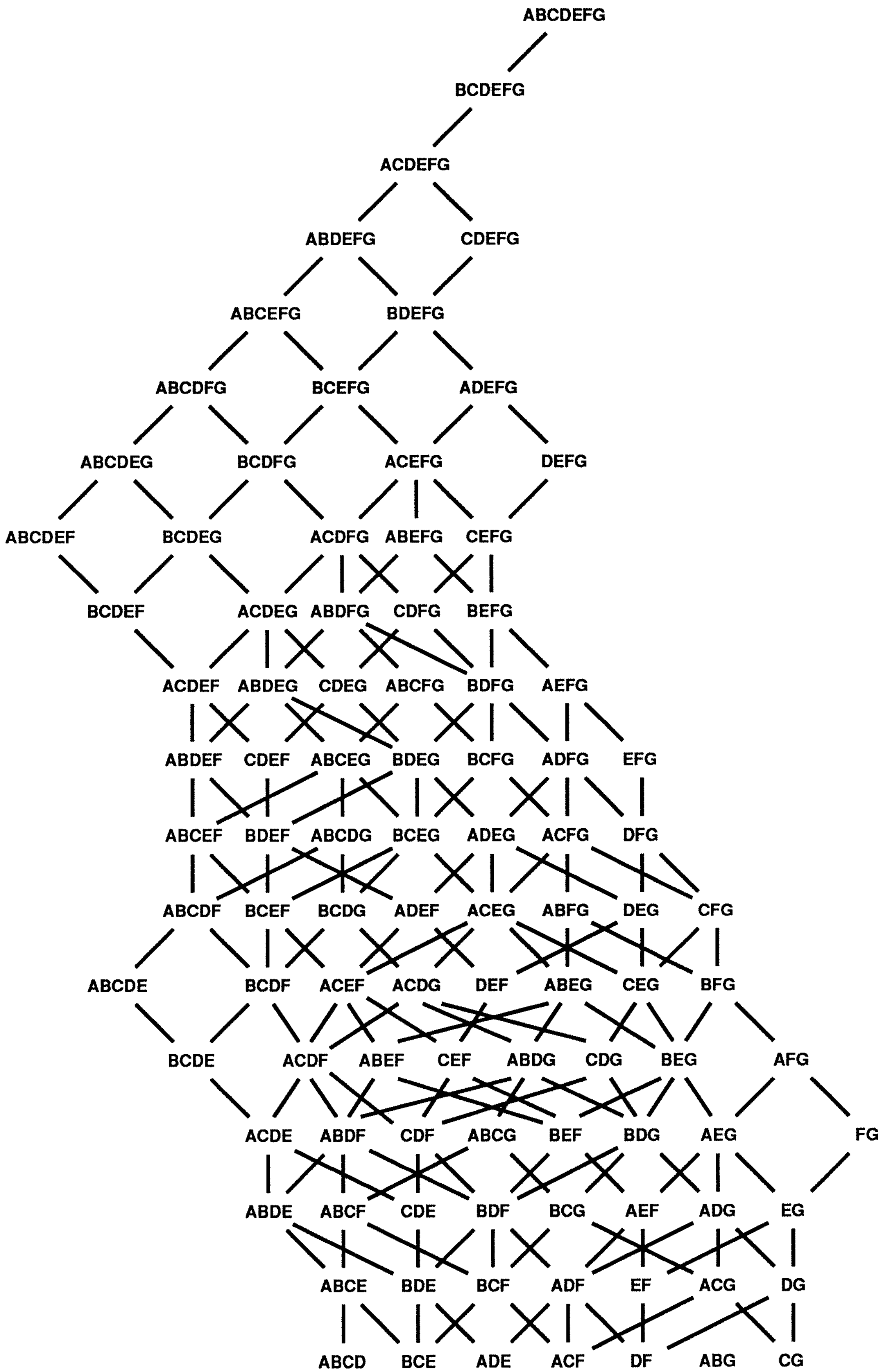






1 < A < B < C < D < E < F < G

For the sake of legibility, in this rendering all the **+** signs have been removed. On the other hand, these posets can be read as products rather than sums, as long as all quantities are greater than **1** instead of **0**.


ABCDEFG
BCDEFG
ACDEFG
ABDEFG CDEFG
ABCEFG BDEFG
ABCDFG BCEFG ADEFG
ABCDEG BCDFG ACEFG DEFG
ABCDEF BCDEG ACDFG ABEFG CEFG
BCDEF ACDEG ABDFG CDFG BEFG
ACDEF ABDEG CDEG ABCFG BDFG AEFG
ABDEF CDEF ABCEG BDEG BCFG ADFG EFG
ABCEF BDEF ABCDG BCEG ADEG ACFG DFG
ABCDF BCEF BCDG ADEF ACEG ABFG DEG CFG
ABCDE BCDF ACEF ACDG DEF ABEG CEG BFG
BCDE ACDF ABEF CEF ABDG CDG BEG AFG
ACDE ABDF CDF ABCG BEF BDG AEG FG
ABDE ABCF CDE BDF BCG AEF ADG EG
ABCE BDE BCF ADF EF ACG DG
ABCD BCE ADE ACF DF ABG CG