

Nov 10, 1975

Dear Neil

I have finally awoken^{ed} to the fact that if
 $g(t) = t a(t)$, $G(t) = t A(t)$ and $g(G(t)) = G(g(t)) = t$
 then, if $a(0) = A(0) = 1$,

$$a_n = B_n(-A_1, \dots, -A_n)$$

$$A_n = B_n(-a_1, \dots, -a_n)$$

with

$$B_n(b_1, \dots, b_n) = \frac{1}{n!} \sum_{k_1, \dots, k_n} \binom{n+k}{n} \frac{k_1! \dots k_n!}{k_1! \dots k_n!} b_1^{k_1} \dots b_n^{k_n}$$

the reversion of series multivariable polynomial Com. Id. p. 139

Here are some consequences for the handbook.

1. $B_n(1, 1, 0, \dots, 0)$ is sequence 1146, Bwt

$$B_n(1, 1, \dots, 0) = \frac{1}{n!} \sum_{k=0}^{n-1} \binom{n-k}{k} \binom{2n-k}{n-k} = \frac{1}{n!} f_n^{(n+1)}$$

$f_n^{(k)}$ the k th convolved Fibonacci C. I, p. 89

2. $B_n(1, 1, \dots, 1) = H_n(2)$, with $H_n(z)$ John Morrison's
 polynomial (C.I., 151), which is sequence 1163

These also the numbers of Schröder's third problem.
 (as already noticed in your references). Incidentally, if

$g(t) = t H(t) = \sum_{n \geq 0} h_n(t) t^{n+1}$, the $G(t) = t(1-2t)(1-t)^{-1}$
 and $G(g(t)) = t \Rightarrow (1+t)H(t) - 2tH^2(t) = 1$, agreeing with
 Motzkin (and Schröder)

3. Somehow or other I started a reexamination of $c(x)$;
 $c(x) = \sum_{n \geq 0} c_n x^n$ $c_n = \frac{1}{n!} \binom{2n}{n}$ - Catalan. The enclosed table is the

2.
result. Notice that the following identifications are made

	1	3	4	5	7	9	11	13
Handbook	577	1130	1415	1682	1866	1981	2047	2104

Of course (C.I. p. 153) $c_n(t) = \frac{t}{2n+1} \binom{2n+1}{n} = \frac{t}{n} \binom{2n-1}{n-1}$
Notation: $C^h(x) = \sum c_n(t) x^n$

My week was made last Monday by a letter from Carlson that Acta Math will publish (next year, of course) my paper "The Blossoming of Schröder's Fourth Problem"

As always, best to you and love to Ann
John

Com. solva Catala, numer.

$$C(n, k) = \sum_{i=0}^k \binom{n}{i}$$

$$C(n, k) = C(n, n-k) \quad C_{n-1, k} = \sum_{i=0}^{k-1} \binom{n-1}{i} = \sum_{i=0}^{n-1-k} \binom{n-1}{i} = \frac{1}{2} \left(\sum_{i=0}^{n-1} \binom{n-1}{i} \right) = \frac{1}{2} (2^{n-1}) = 2^{n-2}$$

Hamblett	10	1	2	3	4	5	6	7	8	9	10
527	1	1	2	5	14	42	132	429	1436	4860	1796
"	2	1	5	14	42	132	429	1436	4860	1796	55786
1136	3	1	9	27	76	247	1001	3539	11934	41946	149321
1415	4	1	14	48	145	575	2082	7192	25194	90441	326876
1662	5	1	20	76	275	1001	3540	13206	48456	177651	653752
1867	6	1	29	116	429	1638	6125	23256	87216	326876	1259955
1866	7	1	35	154	617	2545	9996	38940	149226	572035	2187855
16507	8	1	44	208	916	3502	15504	61916	245159	961400	3748466
1481	9	1	54	273	1220	5505	23256	95931	389287	1562295	6218216
-	10	1	65	353	1716	7952	33915	144210	600375	2760755	10019005
22077	11	1	77	440	2297	10159	43279	211508	904715	3790916	15737815
16507	12	1	90	574	3007	14366	61295	303955	1332045	5720816	2419206
2104	13	1	104	663	3715	19019	90042	42707	192465	845422	3641347
	14	1	119	795							
	15	1	136	950							
	16										
	17	1	154	112	-5	-14	-42	-132	-404	-1230	-4860
	18	1	172	132	-5	-14	-42	-132	-404	-1230	-4860
	19	1	191	154	-3	-8	-28	-90	-297	-100	-3732
	20	1	211	177	-1	-4	-14	-48	-165	-572	-2066
	21	1	232	200	0	-1	-5	-26	-75	-275	-1001
	22	0	254	224	0	0	-1	-6	-27	-110	-429
	23	-1	277	249	0	0	0	-1	-7	-35	-154
	24	-2	302	276	2	6	0	0	-1	-8	-44
	25	-3	329	304	9	6	9	0	0	-1	-9
	26			333	13	15	15	15	15	15	15

669
1628
1190
1859

Planted trees with no pts of degree 2 + root removed
by number of pts + no of endpoints

$(1+x)G(x,y) + x \exp[G(x,y) + G(x,y) + \dots]$
 $Q(x,y) = \sum x^n y^m G_n(x,y)$

GN-553 (12-55)

1628

Row Sum	Column Sum	$Q_{1,0}$	$Q_{0,1}$	1	2	3	6	10	19	35	67	127	248	462	952	1865	3765	7546	15221	30802
$Q_{n,1} = 1$		1	0	1	2	3	6	10	19	35	67	127	248	462	952	1865	3765	7546	15221	30802
$Q_{n,2} = \binom{n+1}{2}$		1	1	3	6	10	19	35	67	127	248	462	952	1865	3765	7546	15221	30802		
$Q_{n,3} = \binom{n+2}{3}$		1	3	6	10	19	35	67	127	248	462	952	1865	3765	7546	15221	30802			
= no. of partitions		1	3	6	10	19	35	67	127	248	462	952	1865	3765	7546	15221	30802			
random		1	3	6	10	19	35	67	127	248	462	952	1865	3765	7546	15221	30802			
$C_{n+1} + I_{n+1}$		1	4	10	28	77	217	616	1771	5052	14328	40176	111711	311485	877193	2446716	6842271	19370752	54776351	155051984
		1	6	15	40	105	274	714	1964	5380	14700	40176	111711	311485	877193	2446716	6842271	19370752	54776351	155051984
		1	8	21	58	154	413	1101	2908	7701	20548	56009	151825	407775	1117193	3014854	8171937	22148541	59476351	162051984
		1	11	28	77	217	616	1771	5052	14328	40176	111711	311485	877193	2446716	6842271	19370752	54776351	155051984	42776351
		1	15	36	96	252	662	1827	5052	14328	40176	111711	311485	877193	2446716	6842271	19370752	54776351	155051984	42776351
		1	21	55	145	392	1050	2908	7901	21771	60176	167711	467193	1294854	3614854	10071937	28148541	77476351	214051984	59476351
		1	28	72	196	513	1401	3828	10408	29080	80176	221711	607193	1694854	4714854	13071937	36148541	10071937	28148541	77476351
		1	36	93	252	662	1827	5052	14328	40176	111711	311485	877193	2446716	6842271	19370752	54776351	155051984	42776351	1190
		1	46	120	325	877	2446	6842	19370	54776	155052	427764	1207193	3314854	9271937	25748541	7271937	20348541	57476351	162051984
		1	59	156	413	1101	3008	8201	22771	63176	177711	497193	1394854	3874854	10871937	30148541	8371937	23476351	65476351	182051984
		1	77	205	546	1501	4058	11171	30771	85076	234711	647193	1814854	5074854	14371937	40148541	11271937	31476351	87476351	244051984
		1	100	274	734	2008	5501	15171	41771	116076	321711	897193	2514854	6974854	19771937	55148541	15471937	43476351	121051984	34476351
		1	132	352	962	2668	7301	20080	55071	153076	427711	1207193	3374854	9474854	26771937	75148541	21171937	59476351	166051984	47476351
		1	171	462	1274	3528	9701	26680	74071	205076	571711	1597193	4474854	12674854	35771937	10148541	28771937	81476351	231051984	65476351
		1	224	616	1701	4701	13008	36071	100076	277711	777193	2174854	6074854	17171937	48148541	13671937	38476351	109051984	30476351	86051984
		1	294	813	2252	6201	17208	47071	130076	360711	1000793	2774854	7774854	21771937	60748541	16971937	47476351	133051984	37476351	106051984
		1	384	1101	3008	8201	22771	63176	177711	497193	1394854	3974854	11271937	31476351	87476351	244051984	68476351	194051984	54476351	154051984
		1	504	1401	3828	10408	29080	80176	221711	617193	1714854	4774854	13471937	37476351	106051984	30476351	86051984	244051984	68476351	194051984
		1	664	1827	5052	14328	40176	111711	311485	877193	2446716	6842271	19370752	54776351	155051984	42776351	121051984	34476351	98476351	284051984
		1	877	2446	6842	19370	54776	155052	427764	1207193	3374854	9474854	26771937	75148541	21171937	59476351	166051984	47476351	136051984	39476351
		1	1164	3252	9001	25008	69071	193076	540711	1530793	4277193	12074854	33771937	94748541	26771937	75148541	21171937	59476351	166051984	47476351
		1	1554	4328	11901	33176	92771	257076	720711	2030793	5774854	16374854	46771937	133748541	37771937	10676351	30476351	86051984	244051984	68476351
		1	2054	5701	15908	44071	123076	340711	960793	2677193	7514854	21171937	59476351	166051984	47476351	136051984	39476351	112051984	32476351	94051984
		1	2744	7701	21771	60176	169711	477193	13474854	37771937	10676351	30476351	86051984	244051984	68476351	194051984	54476351	154051984	44476351	126051984
		1	3644	10408	29080	80176	221711	617193	1714854	4774854	13471937	37476351	106051984	30476351	86051984	244051984	68476351	194051984	54476351	154051984
		1	4844	13308	37001	104076	290711	817193	23474854	66771937	19376351	54776351	155051984	42776351	121051984	34476351	98476351	284051984	82476351	244051984
		1	6444	17701	49008	136071	380793	1060711	3047937	86071937	244051984	68476351	194051984	54476351	154051984	44476351	126051984	36476351	104051984	30476351
		1	8644	23408	64001	180076	500711	1400793	3947937	112051984	32476351	94051984	26476351	76476351	224051984	64476351	184051984	54476351	154051984	44476351
		1	11444	31408	87001	240076	670711	1900793	5347937	154051984	44476351	134051984	39476351	112051984	32476351	94051984	26476351	76476351	224051984	64476351
		1	15144	41408	114001	320076	890711	2400793	6847937	204051984	59476351	166051984	47476351	136051984	39476351	112051984	32476351	94051984	26476351	76476351
		1	20044	55408	154001	430076	1200711	3400793	9647937	274051984	82476351	244051984	68476351	204051984	59476351	166051984	47476351	136051984	39476351	112051984
		1	26844	74408	204001	570076	1600711	4600793	13407937	394051984	112051984	32476351	94051984	26476351	76476351	224051984	64476351	184051984	54476351	154051984
		1	35844	100408	280001	780076	2200711	6300793	18407937	534051984	154051984	44476351	134051984	39476351	112051984	32476351	94051984	26476351	76476351	224051984
		1	47844	136408	380001	1060076	3000711	8400793	24407937	694051984	204051984	59476351	166051984	47476351	136051984	39476351	112051984	32476351	94051984	26476351
		1	63844	186408	520001	1440076	4000711	11400793	32407937	944051984	274051984	82476351	244051984	68476351	204051984	59476351	166051984	47476351	136051984	39476351
		1	85844	256408	720001	2000076	5600711	16400793	46407937	1344051984	394051984	112051984	32476351	94051984	26476351	76476351	224051984	64476351	184051984	54476351
		1	113844	356408	1000001	2800076	7800711	22400793	64407937	1844051984	534051984	154051984	44476351	134051984	39476351	112051984	32476351	94051984	26476351	76476351
		1	151844	496408	1380001	3800076	10600711	30400793	86407937	2444051984	694051984	204051984	59476351	166051984	47476351	136051984	39476351	112051984	32476351	94051984
		1	200844	676408	1900001	5300076	15000711	43400793	124407937	3644051984	1044051984	30476351	86051984	244051984	68476351	204051984	59476351	166051984	47476351	136051984
		1	268844	926408	2600001	7400076	20400711	59400793	174407937	5144051984	1544051984	44476351	134051984	39476351	112051984	32476351	94051984	26476351	76476351	224051984
		1	358844	1266408	3600001	10200076	28400711	82400793	234407937	6944051984	2044051984	59476351	166051984	47476351	136051984	39476351	112051984	32476351	94051984	26476351
		1	478844	1746408	5000001	14000076	39400711	114400793	324407937	9444051984	2744051984	82476351	244051984	68476351	204051984	59476351	166051984	47476351	136051984	39476351
		1	638844	2406408	6900001	19000076	53400711	154400793	464407937	13444051984	3944051984	112051984	32476351	94051984	26476351	76476351	224051984	64476351		

Planted trees with no pts of degree 2 + root removed
by number of pts + no of endpoints

$$(1+x) \phi(x,y) + x \exp[\phi(x,y) + \phi(x,y) + \dots]$$

$$\phi(x,y) = \sum x^i y^j G_{i,j} = \sum x$$

Row	Col	Labels	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
2	1	2	1	2	3	6	10	19	35	67	127	248	462	852	1585	3765	7546	15221	30802								
3	2	3	1	3	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20							
4	3	4	1	4	1	2	2																				
5	4	5	1	1	1	2	3	5	3																		
6	5	6	1	1	1	1	4	10	12	6																	
7	6	7	1	1	1	1	1	5	16	29	28	11															
8	7	8	1	1	1	1	1	1	6	24	57	85	66	23													
9	8	9	1	1	1	1	1	1	7	33	99	192	231	157	46												
10	9	10	1	1	1	1	1	1	8	44	157	382	615	634	98												
11	10	11	1	1	1	1	1	1	9	56	234	682	1380	1380	1905	890											
12	11	12	1	1	1	1	1	1	10	70	333	1133	456	4782	5746												
13	12	13	1	1	1	1	1	1	11	85	456	102	102	1792	16536												
14	13	14	1	1	1	1	1	1	12	102	606	2652	8635	2652	8635												
15	14	15	1	1	1	1	1	1	13	119	786	3819	998	998	3819												
16	15	16	1	1	1	1	1	1	14	140	140	140	140	140	140												
17	16	17	1	1	1	1	1	1	15	15	15	15	15	15	15												
18	17	18	1	1	1	1	1	1	16	161	161	161	161	161	161												
19	18	19	1	1	1	1	1	1	17	170	170	170	170	170	170												
20	19	20	1	1	1	1	1	1	18	180	180	180	180	180	180												
21	20	21	1	1	1	1	1	1	19	190	190	190	190	190	190												
22	21	22	1	1	1	1	1	1	20	200	200	200	200	200	200												
23	22	23	1	1	1	1	1	1	21	210	210	210	210	210	210												
24	23	24	1	1	1	1	1	1	22	220	220	220	220	220	220												

1859 → $G_{n,n-3}$

1860 $G_{n,n-4}$

669

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