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AUG 26 1974

Professor W. Butler
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Dear Bill Butler:

Andy Kalotay showed me your interesting note on inserting parentheses. Actually this is a very old result, first published by the Reverend T. P. Kirkman in the Philosophical Transactions of Royal Society in 1857, Vol. 147, pp. 217-272, (without proof); and first proved by Cayley in 1891, in Proc. London Math. Soc. 22, pp. 237-262. But so many people have worked on this and similar problems that it is hard to assign priorities.

Of course there are three equivalent problems (i) inserting parentheses, (ii) dissecting a polygon, (iii) counting planar trees. For a proof that these are equivalent see §3.5, pp. 18-20 of my book "A Handbook of Integer Sequences" (Academic Press, N.Y., 1973). Actually I only show the correspondence between the three problems in the case of complete dissections (r = n-1), but the general correspondence is then obvious. The latter is also given in I. M. H. Etherington, Some problems of non-associative combinations, Edinburgh Math. Notes, No. 32, 1940, pp. i-vi.

For some of the hundreds of references on these problems see for example:

W. G. Brown, Historical note on a recurrent combinatorial problem, Am. Math. Monthly 72 (1965), pp. 973-977 and

H. W. Gould, Research Bibliography of Two Special Number Sequences, Mathematica Monongaliae 12 (1971).

Yours, sincerely,

MH-1216-NJAS-mv

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Nov. 3, 1974

Dr. Neil Sloane
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Dear Neil,

Thank you for your letter of Aug. 26 on my note on parentheses. It was very helpful although by the time I received it I had spoken to W. G. Brown who was able to put me on track. Enclosed is a bibliography of all the references to Kirkman's result known to me – if you know of any others I would be very grateful to hear of them. In particular I wish to know if any proofs of the result are known or have appeared besides those of Cayley (1891) and Watson (1962).

What amazes me about the problem is that the history of the generalization bears no ressemblance whatever to that of the original problem. Pursuing this, I have been able to generalize several of the classical proofs (in particular Rodrigue's very elegant one involving no analysis) obtaining proofs all of which are simpler than either that of Cayley (depending as it does on Lagrange's almost forgotten formula) or that of Watson.

Thank you for your prompt response to my original flier.

Yours sincerely,
Bill Butler

Bibliography for Kirkman's Generalization of the Problem of Euler-Segner-Fuss-Catalan. (On the number of divisions of a convex r-gon by k non-intersecting diagonals, $0 \le k \le r-3$.)

(Gould's Bibliography)

1. T. P. Kirkman, On autopolar polyhedra, Phil. Trans. Royal Soc. London 147 (1857) 183-215

#114 2. T. P. Kirkman, On the k-partitions of the r-gon and r-ace, Phil. Trans. Royal Soc. London 147 (1857) 217-272

#42 3. A. Cayley, Note on the summation of a certain factorial expression, Phil. Mag. (4) 13 (1857) 419-423

#163 4. P.M.E. Prouhet, Problem #774 Nouv. Ann. Math. (2) 5 (1866) 384

#131 5. E. Lucas, Théorie des Nombres, vol. I, Gauthier-Villars, Paris, 1891

#44 6. A. Cayley, On the partitions of a polygon, Proc. London Math. Soc. (1) 22 (1891), 237-262

#206 7. G. N. Watson, A proof of Kirkman's hypothesis, Proc. Edinburgh Math. Soc. (2) 13 (1962) 131-138

8. D. DeFongeres and D. Tamari, Polyhedral structure of bracket complexes, Notices of the Am. Math. Soc. 19 (1972) A31